

## Routing and Congestion Control for Ad-hoc Networks

Mohsen Shafieirad<sup>1</sup> and Masoud Shafiee<sup>1</sup>

Computer Science Department  
Islamic Azad University Iran

**Abstract:** In this paper, we consider the mobility effect with a jointly optimal design of cross-layer congestion control, routing and scheduling for ad-hoc networks. We first formulate the rate constraint and scheduling constraint. In this way, we use multi-commodity flow variables. Then formulate resource allocation in networks with fixed wireless channel and single-rate devices. Because of entrance of the effect mobility in optimal design, we formulate resource allocation as utility and cost function, together in a maximization problem with those constraints. By dual decomposition, decompose the resource allocation problem vertically into three sub-problems: congestion control, routing and scheduling. These three sub-problems interact through congestion and link price.

**Keywords:** Mobile Ad-hoc Networks; cross-layer design; distributed algorithm; mobility.

### 1. Introduction

Traditionally, network protocols take a layered structure and perform congestion control, routing and scheduling independently at different layers. However, wireless spectrum is a scarce resource, and it has importance to use the wireless channel efficiently. For this purpose, congestion control, routing and scheduling should be jointly designed.

The need for joint design across these three layers is motivated by three observations. First, wireless channel is a shared medium and interference-limited. In wire-line networks, links are disjoint resources with fixed capacities. But in ad-hoc wireless networks the link capacities are “elastic” and because of the contention among links we have a constraint for resource allocation, i.e., they determine the feasible rate region at link layer (Ren & Zhang, 2010) and (Korger et al., 2010). Second, most routing schemes for ad hoc networks select paths that minimize hop count. Therefore, a route is defined for any source-destination pair, independent of traffic, interference and contention among links. This may result in congestion at some region while other regions are not utilized perfectly and optimally. To use the wireless spectrum more efficiently, we should exploit multiple paths based on the pattern of traffic demand, interference and contention among links. As we will see, routing is then determined from the rate and scheduling constraints. Lastly, TCP congestion control can be interpreted as distributed algorithms (primal-dual problem) to maximize aggregate utility. In these algorithms, a network is assumed with fixed link capacities and pre-specified routes.

In all cross-layer designs for congestion control in ad-hoc networks, the effect of node mobility on source rate control and resource allocation is not considered. Mobility can effect on link cost (for example the length of link). When nodes have mobility, the distances between them, change frequently. Consequently, the path for transmitting data will become longer or shorter. If in congestion control and routing, we don't consider the mobility effect, and just consider the congestion price, the price that will be imposed on us when occurring congestion in network; data may be transmitted from longer path. It is obvious that this problem is not optimal. Increasing length between two nodes will cause the larger delay to transmit data and to attenuate the radio signal. When the radio signal decreases, the transmission error, data loss and interference increase.

Here we extend the basic utility maximization formulation to handle the link cost simultaneously. We model the mobility effect as link cost and by this extension, especially in ad-hoc networks without pre-specified routes, we are able to handle the mobility in congestion control and solve

the maximization problem. We model the contention between wireless links as a contention flow graph. By this structure, we indicate which sets of links interfere and cannot transmit simultaneously. The feasible rate region at link layer is the convex hull of corresponding rate vectors of independent sets of the contention flow graph. We use multi-commodity flow variables (Chen et al., 2006) to formulate rate constraint at the network layer and formulate resource allocation in wireless ad hoc networks with fixed channel or single-rate devices as a maximization problem with those constraints. In this maximization problem, by introducing a new multi-commodity variable for link cost, we consider both utility and cost in optimization problem. We then apply duality theory to decompose the system problem vertically into congestion control sub-problem and routing/scheduling sub-problem. These two sub-problems interact through congestion prices and link costs. Based on this decomposition, by using sub-gradient method we can obtain a distributed algorithm for link state aware cross-layer design for joint congestion control, routing and scheduling and prove the convergence of this algorithm to optimal value. In this design, the source adjusts its sending rate according to congestion price generated locally at the source node. By differential price of neighboring nodes, that is obtained with congestion price and link cost, the optimal scheduling and routing is satisfied.

## 2. Related Works

The work in (Kelly et al., 1998), (Low & Lapsley, 1999) and (Low, 2003) provides a utility-based optimization framework for internet congestion control. For ad-hoc networks, similar framework has been applied for congestion control, (Chiang, 2005), (Oh et al., 2009), (Xue et al., 2003) and (Chen et al., 2005). In (Chen et al., 2005), the authors study joint congestion control and media access control for ad-hoc wireless network, and formulate rate allocation as a utility maximization problem with the constraints that arise from contention for channel access.

In (Neely et al., 2005), the authors use multi-commodity flow variables to characterize the network capacity region for a wireless network with time-varying channel, and propose a joint routing and power allocation policy to stabilize the system whenever the input rates are within this capacity region. In (Jain et al., 2003), the authors study the impact of interference on multi-hop wireless network performance. They model wireless interference using the conflict graph. We use the same structure to model the contention among wireless links. In (Hajek & Sasaki, 1988), the author uses a similar model to study the problem of jointly routing the flows and scheduling the transmissions to determine the achievable rates in wireless networks. These works focus on the interaction between link and network layers and characterize the achievable rate region at network layer. In (Chen et al., 2006), the authors include the end-to-end transport layer, and so, the network uses congestion control to automatically explore the achievable rate region while optimizing some global objective for the end users.

Motivated by the duality model of TCP/AQM, which is an example of “horizontal” decomposition via dual decomposition; researchers have extended the utility maximization framework to provide a general cross-layer design methodology. Duality theory leads to a natural “vertical” decomposition into separate designs of different layers that interact through congestion price. Recent works along this line of “layering as optimization decomposition” (Chiang et al., 2006) includes (Xiao et al., 2004) for routing and resource allocation, (Chen et al., 2006), for TCP and physical layer, and (Chen et al., 2005), (Lee et al., 2006), (Lin & Shroff, 2004) for joint TCP and media access control or scheduling. In above cross-layer designs, the node mobility has not been considered. We present a method to handle this problem by defining a link cost variable.

Our goal in this paper is to import the node mobility effect on congestion control in ad-hoc networks via a cross-layer design. In this way, we use the main idea presented in (Chen et al., 2006). In (Chen et al., 2006), a cross-layer joint design for congestion control, routing and scheduling is presented. We extend the work to handle the mobility problem and study its effect simultaneously, in congestion control, routing and scheduling.

### 3. The Model

Consider an ad hoc wireless network with a set  $N$  of nodes and a set  $L$  of links. These links are directed and symmetric, i.e., link  $(j, i) \in L$  if and only if  $(i, j) \in L$ . The topology of network is static. Each link  $l \in L$  has a fixed finite capacity  $c_l$  bits per second, i.e., we assume that the wireless channel is fixed. In order to thoroughly simulate a new protocol for an ad-hoc network, it is imperative to use a mobility model that accurately represents the mobile nodes (MNs). Currently, there are two types of mobility models used in the simulation of networks: traces and synthetic models (Korger et al., 2010), (Sanchez & Manzoni, 1999) and (Eason et al., 1955). Traces are those mobility patterns that are observed in real-life systems. However, new network environments such ad-hoc networks, are not easily modeled if traces have not yet been created. In this type of situation, it is necessary to use synthetic models. Synthetic models attempt to realistically represent the behaviors of MNs without the use of traces. We will use a proper synthetic mobility model for movement of nodes in wireless ad-hoc network. Based on this model, we define a cost variable for each link in wireless network.

Wireless channel is a shared medium and interference-limited where links contend with each other for exclusive access to the channel. We will use the conflict graph to capture the contention relations among links. The feasible rate region at link layer is then a convex hull of the corresponding rate vectors of independent sets of the conflict graph. We will introduce multi-commodity cost variables, which correspond to the link cost to describe the mobility effect on link length and multi-commodity flow variables to the capacities allocated to the flows towards different destinations, to describe the rate constraint at network layer. The resource allocation is formulated as a utility minus cost maximization problem with schedulability and rate constraints.

#### 3.1. Mobility Model

We assume a military environment and according to this assumption use a mobility model in this paper. We assume the nodes remain in each other radio range and because of mobility, the connectivity of network is preserved. In military movements, like movement of soldiers, tanks and etc, nodes have continuous movement and this movement is not quite random, because movements are based on human behaviors and military strategies. In other words, the current move in new step is not independent of its past move and depends on it, both in speed and direction. By allowing past speeds (and directions) to influence future speeds (and directions), we can eliminate the sudden stops and sharp turns, like realistic movements (Camp et al., 2002).

According to above discussion, we use following model for mobility. A velocity vector  $\vec{v} = (v, \theta)$  is used to describe a MNs' velocity  $v$  as well as its direction  $\theta$ ; the MNs' position is represented as  $(x, y)$ . Both the velocity vector and the position are updated at every  $\Delta t$  time steps according to the following formulas:

$$v(t + \Delta t) = \min\{\max[v(t) + \Delta v, 0], V_{max}\} \quad (1)$$

$$\theta(t + \Delta t) = \theta(t) + \Delta\theta;$$

$$x(t + \Delta t) = x(t) + v(t) * \cos\theta(t);$$

$$y(t + \Delta t) = y(t) + v(t) * \sin\theta(t);$$

where  $V_{max}$  is the maximum velocity defined in the simulation,  $\Delta v$  is the change in velocity that is uniformly distributed between  $[-A_{max} * \Delta t, A_{max} * \Delta t]$ ,  $A_{max}$  is the maximum acceleration of a given MN,  $\Delta\theta$  is the change in direction that is uniformly distributed between  $[-\alpha * \Delta t, \alpha * \Delta t]$  and  $\alpha$  is the maximum angular change in the direction a MN is traveling (Haas, 1997). In problem formulation, we introduce multi-commodity variables as the amount of costs that link incurred while transmitting flow to the destination  $k$ , like definition used for flow variables in (Chen et al., 2006).

#### 3.2. Schedulability and Rate Constraint

In this paper, we consider a network with primary interference model: links that share a common node cannot transmit or receive simultaneously, but links that do not share nodes can do so. Under this interference model, we can construct a conflict graph (Jain et al., 2003) that shows the contention relations among the links. In the conflict graph, each vertex represents a link, and an edge between two vertices denotes the contention between the two corresponding links: these links cannot transmit at the same time. Figure 1 shows an example of a wireless ad-hoc network and its conflict graph with primary interference model.

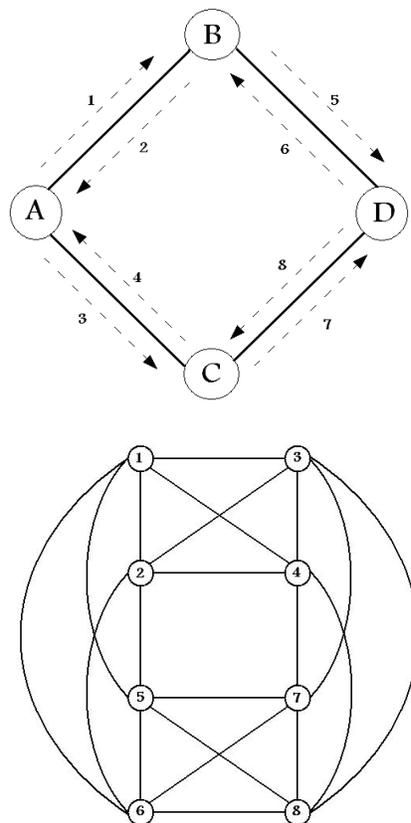


Figure 1. An ad hoc wireless network (upper) with 4 nodes and 8 links and its conflict graph (lower)

By a conflict graph, we identify all its independent sets of vertices. It's obvious that all links in an independent set can transmit simultaneously. For instance, in Fig.1, {1, 8}, {2, 8}, {4, 6} ... are some independent sets. Let E denote the set of all independent sets. We represent an independent set by e as a  $|L|$ -dimensional rate vector  $r^e$ , where the  $l$  th entry is:

$$r_l^e := \begin{cases} c_l & \text{if } l \in e \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Then define the feasible rate region  $\Pi$  at the link layer as a convex hull of these rate vectors:

$$\Pi := \{r: r = \sum_e a_e r^e, a_e \geq 0, \sum_e a_e = 1\} \quad (3)$$

Thus, for a link flow vector such y, the schedulability constraint says that  $y \in \Pi$ .

Denote the set of destination nodes of network layer flows by D.

Let  $f_{i,j}^k \geq 0$  denote the amount of capacity of link  $(i,j)$  allocated to the flow to destination  $k$ . Then  $f_{i,j} = \sum_{k \in D} f_{i,j}^k$  is the aggregate capacity on link  $(i,j)$ . From the schedulability constraint,  $f = \{f_{i,j}\}$  should satisfy the condition:

$$y \in \Pi \quad (4)$$

Let  $x_i^k \geq 0$  denote the flow generated at node  $i$  towards destination  $k$ . Then the aggregate capacity for its incoming flows and generated flow to the destination  $k$  should not exceed the summation of the capacities for its outgoing flows to  $k$ :

$$x_i^k \leq \sum_{j:(i,j) \in I} f_{i,j}^k - \sum_{j:(i,j) \in I} f_{j,i}^k \quad (5)$$

$$i \in N, k \in D, i \neq k$$

Equation (5) is the rate constraint for resource allocation. Results in (Chen et al., 2006) give multi-commodity flow variables a different interpretation as the amount of link capacities allocated to the flows of different destinations.

### 3.3. Problem Formulation

We use  $l \in L$  or alternatively node pair  $(i,j) \in N \times N$  to denote a link. Assume the network is shared by a set  $S$  of sources indexed by  $s$ . We use node pair  $[i,k] \in S \times D$  to denote a network layer flow.

Assume each source  $s$  attains a utility  $U_s(x_s)$  when it transmits at rate  $x_s$  packets per second. We assume  $U_s(\cdot)$  is continuously differentiable, increasing, and strictly concave.

For cost function, we introduce the multi-commodity link cost variable for each link. According to the mobility model in section 3.1 we model the cost mobility with cost link. For each link we set a cost. This cost is dependent to length of that link. For example, if due to mobility of nodes, the length of link increases, consequently its cost, increases too. Because of transmitting data by this link, we see a larger path and a larger delay. Thus, we use  $\lambda_{i,j}^k$  for link cost. It is interpreted in this way: the link  $(i,j)$  to transmit data  $f_{i,j}^k$  to destination  $k$ , incurs the cost  $\lambda_{i,j}^k$ . Our objective is to choose source rates  $x_s$  and allocated capacities  $f_{i,j}^k$  so as to solve the following global problem:

$$\max_{x_s \geq 0, f_{i,j}^k \geq 0} (\sum_s U_s(x_s) - \sum_{(i,j),k} \lambda_{i,j}^k f_{i,j}^k) \quad (6)$$

$$x_i^k \leq \sum_{j:(i,j) \in L} f_{i,j}^k - \sum_{j:(i,j) \in L} f_{j,i}^k \quad (7)$$

$$f \in \Pi \quad (8)$$

where  $i \in N, k \in D, i \neq k$ , and  $x_i^k = 0$  if  $[i,k] \notin S \times D$ .

Solving the system problem (6)-(8) directly requires coordination among possibly all sources and links, thus is impractical in real network. Since (6) is a convex optimization problem with strong duality, distributed algorithms can be derived by formulating and solving its Lagrange dual problem. In the next section, we will solve the dual problem and interpret the resulting algorithm in the context of joint design of congestion control, routing and scheduling.

### 4. Cross-Layer Design via Dual Decomposition

Consider the Lagrangian problem with respect to rate constraints:

$$L(p, x, f) = \sum_s U_s(x_s) - \sum_{(i,j),k} \lambda_{i,j}^k f_{i,j}^k - \sum_{i \in N, k \in D, i \neq k} p_i^k (x_i^k - \sum_{j:(i,j) \in L} f_{i,j}^k - \sum_{j:(i,j) \in L} f_{j,i}^k) \quad (9)$$

The dual problem to the primal problem (6-8) is:

$$\min_{p \geq 0} D(p) \quad (10)$$

With partial dual function:

$$D(p) = \max(L(p, x, f)) \quad (11)$$

$$f \in \Pi$$

where we relax only the constraint (7) by introducing Lagrange multiplier  $p_i^k$  for node  $i$  and destination  $k$ . The maximization problem in (11) can be decomposed into the following two sub problems:

$$D_1(p) = \max_{x_s \geq 0} \sum_s U_s(x_s) - \sum_s x_s p_s \quad (12)$$

$$D_2(p) = \max_{f_{i,j}^k \geq 0} \sum_{i \in N, k \in D, i \neq k} p_i^k \left( \sum_{j:(i,j) \in L} f_{i,j}^k - \sum_{j:(i,j) \in L} f_{j,i}^k \right) - \sum_{(i,j),k} \lambda_{i,j}^k f_{i,j}^k \quad (13)$$

If we interpret  $p_i^k$  as the congestion price, the first sub problem is congestion control (Low & Lapsley, 1999) and (Low, 2003), and the second one is the joint routing and scheduling since to solve it we need to determine the amount of capacity  $f_{i,j}^k$  that link  $(i, j)$  is allocated to transmit the data flow towards destination  $k$ . Thus, by dual decomposition, the flow optimization problem decomposes into separate “local” optimization problems of transport and network/link layers, respectively, and they interact through congestion prices. The congestion control problem (12) admits a unique maximizer:

$$x_s(p) = U_s^{-1}(p_s) \quad (14)$$

which adjusts the source rate according to the congestion price of the source node. In contrast to traditional TCP congestion control where the source adjusts its sending rate according to the aggregate price along its path, in our algorithm the congestion price is generated locally at the source node.

We can simply, verify that:

$$\sum_{i,k} p_i^k \left( \sum_{j:(i,j) \in L} f_{i,j}^k - \sum_{j:(j,i) \in L} f_{j,i}^k \right) = \sum_{i,j,k} f_{i,j}^k (p_i^k - p_j^k - \lambda_{i,j}^k) \quad (15)$$

for  $i = k: p_i^k = 0$

Problem (13) is equivalent to the following problem:

$$D_2(p) = \max_{f \in \Pi} \sum_{i,j} f_{i,j} \max_k (p_i^k - p_j^k - \lambda_{i,j}^k) \quad (16)$$

This motivates the following joint scheduling and routing algorithm:

### Algorithm 1: Joint Scheduling & Routing Algorithm

- a) For each link  $(i, j)$ , find destination  $k^*$  such that  $k(t) \in \arg \max_k (p_i^{k(t)}(t) - p_j^{k(t)}(t) - \lambda_{i,j}^{k(t)})$  and define:

$$\omega_{ij}(t) = p_i^{k(t)}(t) - p_j^{k(t)}(t) - \lambda_{i,j}^{k(t)} \quad (17)$$

- b) Scheduling: choose  $\tilde{f}_{ij}$  such that:

$$\tilde{f}(t) \in \arg \max_{f \in \Pi} \sum_{(i,j) \in L} \omega_{i,j}(t) f_{i,j} \quad (18)$$

The scheduling (18) is a difficult problem form ad hoc wireless network. We will discuss its solution in detail in section 4.2.

- c) Routing: over link  $(i, j)$ , send an amount of bits for destination  $k^*$  according to the rate determined by the above scheduling. The  $\omega_{i,j}$  values represent the maximum differential congestion price of destination  $k$  packets between nodes  $i$  and  $j$ . Note that 1–3 is equivalent to solve the problem (13) by the following assignment:

$$f_{ij}^k(t) = \begin{cases} \tilde{f}_{ij}(t) & \text{if } k = k(t) \\ 0 & \text{if } k \neq k(t) \end{cases} \quad (19)$$

Now we come to solve the dual problem (10). Note that the dual function  $D(p)$  is not differentiable, as  $D_2(p)$  is a piecewise linear function and not differentiable. Therefore, we cannot use the usual gradient methods; we will instead solve the dual problem using sub gradient method. Suppose  $f_{ij}^k$  is the solution from the above joint routing and scheduling algorithm. It is easy to verify that:

$$g_i^k(p) = \left( \sum_{j:(i,j) \in L} f_{i,j}^k(p) - \sum_{j:(j,i) \in L} f_{j,i}^k(p) \right) - x_i^k(p) \quad (20)$$

is a sub gradient of dual function  $D(p)$  at point  $p$ . Thus, by the sub gradient method (Bertsekas, 1999), we obtain the following algorithm for price adjustment for node destination pair  $(i, k)$ :

$$p_i^k(t+1) = \left[ p_i^k(t) + \gamma_t \left( x_i^k(p(t)) - \left( \sum_{j:(i,j) \in L} f_{i,j}^k(p(t)) - \sum_{j:(j,i) \in L} f_{j,i}^k(p(t)) \right) \right) \right]^+ \quad (21)$$

Where  $\gamma_t$  is a positive scalar step size, and '+' denotes the projection onto the set  $\mathcal{R}^+$  of non-negative real numbers.

Equation (21) says that, if the demand  $x_i^k(p(t))$  for bandwidth at node  $i$  for the flow to destination  $k$  exceeds the effective capacity  $\sum_{j:(i,j) \in L} f_{i,j}^k - \sum_{j:(j,i) \in L} f_{j,i}^k$ , the price  $p_i^k$  will rise, which will in turn decrease the demand (see (14)) and increases effective capacity (see (16)). Also, note that (21) is distributed and can be implemented by individual nodes using only local information. The above dual algorithm motivates a joint congestion control, routing and scheduling design where at the transport layer sources  $s$  individually adjust their rates according to the local congestion price, and nodes  $i$  individually update their prices according to (21); and at the network/link layer nodes  $i$  solve the scheduling (18) and route data flows accordingly.

#### 4.1. Convergence Analysis

In this subsection, we discuss about convergence property of Algorithm 1. The new iteration may not improve the dual cost for all values of the step size. Because, sub gradient may not be a direction of descent, but makes an angle less than 90 degrees with all descent directions. It is shown that (Bertsekas, 1999), for constant step size, the algorithm is guaranteed to converge to a neighborhood of the optimal value. For diminishing step size, the algorithm is guaranteed to converge to the optimal value. We would like a distributed implementation of the sub gradient algorithm, and thus a constant step size  $\gamma_t = \gamma$  is more convenient. Note that the dual cost usually will not monotonically approach the optimal value, but wander around it under the sub gradient algorithm. The usual criterion for stability and convergence is not applicable. The convergence analysis of our algorithm is exactly similar to the proof in (Chen et al., 2006).

#### 4.2. Scheduling over Ad Hoc Networks

Scheduling over ad hoc network is a difficult problem and in general NP-hard. To see this, note that problem (18) is equivalent to a maximum weight independent set problem over the conflict graph, which is NP-hard for general graphs. However, with the primary interference model we show that problem (18) can be reduced to the maximum weighted matching problem, which is polynomial time solvable.

The scheduling problem (18) is to maximize the weighted sum of the link capacities with the schedulability constraint. It is defined on a weighted digraph whose link weights  $\omega_{i,j}$  can take negative value. To see how it is related to the maximum weighted matching problem, first note that  $\omega_{i,j} > 0$  if  $\omega_{j,i} < 0$  and vice versa. Second, note that links  $(i,j)$  and  $(j,i)$  mutually  $(i,j), (j,i) \in L$ , define an undirected link  $\langle i,j \rangle$  with weight  $\omega'_{i,j} = \max\{\omega_{i,j}c_{i,j}, \omega_{j,i}c_{j,i}\}$ . A matching in a graph is a subset of links, no two of which share a common node. The weight of a matching is the total weight of all its links.

A maximum weighted matching in a graph is a matching whose weight is maximized over all matchings of the graph. Interfere and have the same interference/contention relations with other links. Corresponding to each directed link pair

Let  $L'$  denote the set of undirected links and  $W'$  the corresponding set of weights, the scheduling problem (18) is then equivalent to the maximum weighted matching problem on the weighted graph  $G' = (N, L', W')$ . Note that an (maximal) independent set in the conflict graph will correspond to a (maximal) matching in this undirected graph. Maximum weighted matching problem can be computed in polynomial time (see, e.g., (Papadimitriou & Steiglitz, 1998)), but this requires centralized

implementation. If implemented over an ad hoc network, each node needs to notify the central node of its weight and local connectivity information such that the central node can reconstruct the network topology as a weighted graph. This will lead to an immense communication overhead which is expensive in time and resources. There also exist simpler greedy sequential algorithms to compute a weighted matching at most a factor of 2 away from the maximum (see. e.g. (Preis, 1999)). But they also require centralized implementation. We seek a distributed algorithm where each node participates in the computation itself using only local information.

In (Hoepman, 2004), the author presents a simple distributed algorithm to compute a weighted matching at most a factor of 2 away from the maximum in linear running time  $O(|L'|)$ . This algorithm is a distributed variant of the sequential greedy algorithm presented in (Preis, 1999). We utilize this algorithm to solve our scheduling problem (18) distributedly, as summarized below.

### Algorithm 2: Distributed Scheduling Algorithm

Each node  $i$  carries out the following steps:

**Step1)** Calculate weight  $\omega'_{i,j} = \max\{\omega_{i,j}c_{i,j}, \omega_{j,i}c_{j,i}\}$  for each directed link pair  $(i,j), (j,i) \in L$  incident upon it. Ties are broken randomly.

**Step2)** Find node  $j^*$  such that  $\omega'_{i,j^*}$  is maximized over all links  $\langle i,j \rangle \in L'$  with free neighbors  $j$ .

–If having received a matching request from  $j^*$ , then link  $\langle i,j^* \rangle$  is a matched link. Node  $i$  sends a matched reply and a drop message to all other free neighbors.

–Otherwise, node  $i$  sends a matching request to node  $j^*$ .

**Step3)** Upon receiving a matching request from neighbor  $j$ :

–If  $i = j^*$ , then link  $\langle i,j \rangle$  is a matched link. Node  $i$  sends a matched reply to node  $j$  and a drop message to all other free neighbors.

–If  $j \neq j^*$ , node  $i$  just stores the received message.

**Step4)** Upon receiving a matched reply from neighbor  $j$ , node  $i$  knows link  $\langle i,j \rangle$  is a matched link, and send a drop message to all other free neighbors.

**Step5)** Upon receiving a drop message from neighbor  $j$ , node  $i$  knows that  $j$  is in a matched link, and excludes  $j$  from its freeneighbors set.

**Step6)** If node  $i$  is in a matched link or has no free neighbors, no further action is taken. Otherwise, it will repeat Steps 2-5.

**Step7)** Matched links are allowed to transmit. Nodes  $i, j$  in a matched link  $\langle i,j \rangle$  will schedule the directed link, which gives value  $\omega'_{i,j}$ , to transmit.

Steps 2-6 are the distributed algorithm for maximum weighted matching problem. A link that has been chosen to be in the matching is called matched link. Nodes that are not incident upon any matched link are called free. A matching request is sent to inquire the possibility to choose the link with a neighbor as a matched link. A matched reply is sent to confirm that the link with a neighbor is matched. A node sends drop message to tell its neighbors that it is not free anymore.

Define a link  $\langle i,j \rangle$  to be locally heaviest link if for both  $i$  and  $j$ , its weight is maximized over all links with free neighbors. We can see that this algorithm selects locally heaviest links as matched. Thus, Algorithm 2 is a locally optimal scheduling.

## 5. Numerical Example

In this section, we provide numerical examples to complement the analysis in the previous sections. We consider the network shown in Fig.1, and assume that there are two network layer flows  $A \rightarrow D$  and  $B \rightarrow C$  with the same utility  $U_s(x_s) = \log x_s$ . We have chosen such a small and simple topology to facilitate detailed discussion of the results.

We assume that links CD, DC, AC, CA have one unit of capacity and all other links have 2 units of capacity when active. We first simulate Algorithm 1 with perfect scheduling and without any cost, i.e.  $\lambda_{ij}^k = 0$ . Figures 2 and 3 show the evolution of source rate flows  $A \rightarrow D$  and  $B \rightarrow C$  with step size  $\gamma = 0.1$ . We see that they converge quickly to a neighborhood of the optimal and oscillate around the optimal.

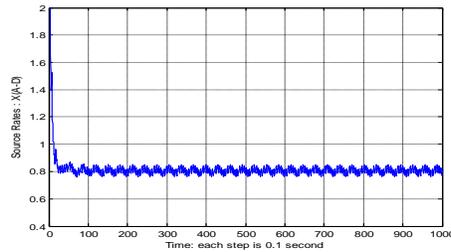


Figure 2. Source rate for flow  $A \rightarrow D$

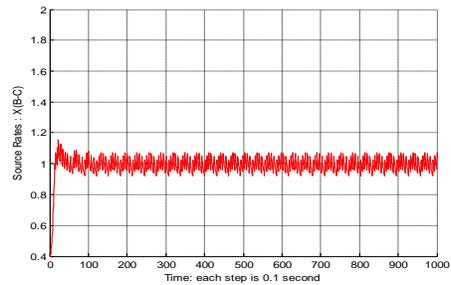


Figure 3. Source rate for flow  $B \rightarrow C$

This oscillating behavior mathematically results from the non-differentiability of the dual function and physically is due to the scheduling process.

However, Figures 4 and 5 show that the average source rates and congestion prices are smooth and approach the optimum value, monotonically.

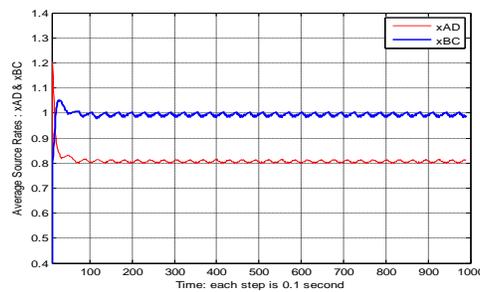
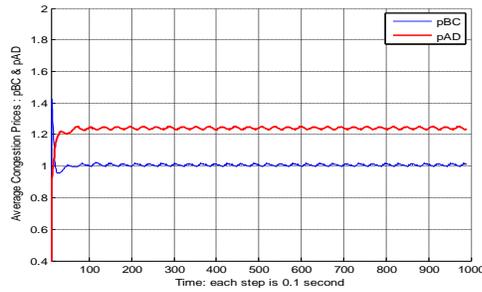


Figure 4. Average source rates



**Figure 5. Average congestion prices**

Tables 1 and 2 show the average link rates allocated to each flow. In all tables in this section, the first column is the sending nodes and the first row is the receiving nodes of each directed link.

Table 1. Average rates of flowB → C through different links with Algorithm 1 (Perfect Scheduling)

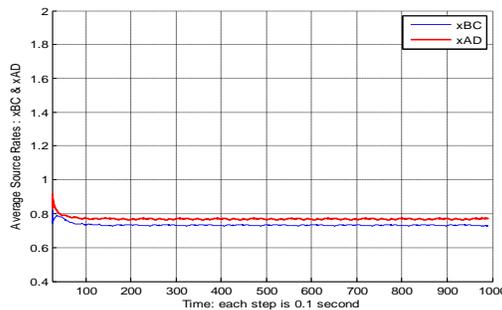
Rates	A	B	C	D
A	0	0	0.1958	-
B	0.2062	0	-	0.7992
C	0	-	0	0
D	-	0	0.8322	0

Table 2. Average rates of flowA → D through different links with Algorithm 1 (Perfect Scheduling)

Rates	A	B	C	D
A	0	0.0080	0.8002	-
B	0	0	-	0
C	0	-	0	0.8012
D	-	0	0	0

From Tables 1 and 2, we can say which paths each flow has used. Note that link BA is not used. This is due to the fact that BA is near the sources and is the link with most contention. So, an optimal routing and scheduling will not activate it.

Now, we simulate Algorithm 1 with the distributed, approximate scheduling (Algorithm 2). The results are shown in Figures 6 and 7. The evolutions of source rates, congestion prices and their averages are similar to those with perfect scheduling: they converge quickly to a neighborhood of stable values.



**Figure 6. Average source rates, distributed scheduling**

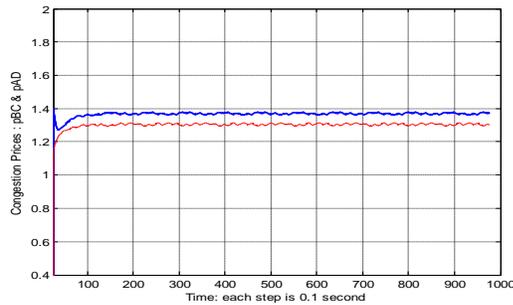


Figure 7. Average congestion prices, distributed scheduling

As expected, the source rates are less than those achieved with perfect scheduling, since the feasible rate region is smaller under the approximate scheduling.

Tables 3 and 4, summarize the average link rates allocated to each flow when we use the distributed algorithm. It is obvious that the routing pattern has been changed, due to the distributed scheduling. Also note that every link is used in routing, since each link has a chance to be a locally heaviest link.

Table 3. Average rates of flows B → C through different links, Algorithm 1 (Distributed Scheduling)

Rates	A	B	C	D
A	0	0	0.3626	-
B	0.3689	0	-	0.3736
C	0	-	0	0
D	-	0	0.3676	0

Table 4. Average rates of flows A → D through different links, Algorithm 1 (Distributed Scheduling)

Rates	A	B	C	D
A	0	0.6314	0.1369	-
B	0	0	-	0.6254
C	0	-	0	0.1329
D	-	0	0	0

Though its worst case performance bound is 1/2, our results show that the degradation of the performance of Algorithm 1 with distributed scheduling is small. Beside its low communication overhead, fast convergence and good performance with distributed scheduling, our cross-layer design is promising for practical implementation.

Now, based on presented model in section 3 for mobility of nodes and for cost function respect to mobility, we simulate Algorithm 1. We assume each node can move in a rectangular area (300,600) and have the mobility pattern for 10 seconds as shown in Figure 8.

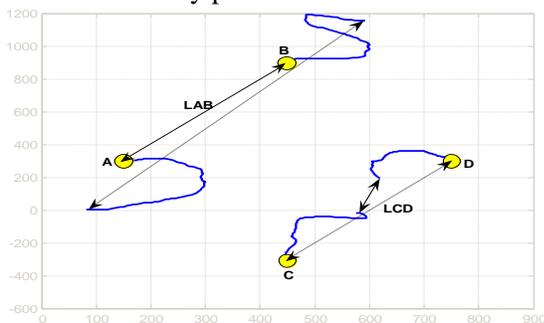


Figure 8. Mobility pattern for 10 seconds

In Figure 9 we see the distance changes between nodes in 10 seconds, too. These distances are link costs in our algorithm.

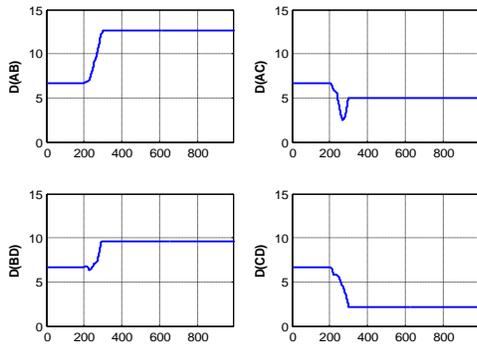


Figure 9. Distance changes between nodes in 10 seconds

Figures 10, 11, 12 and 13 show the source rates and congestion prices when the distances between nodes change.

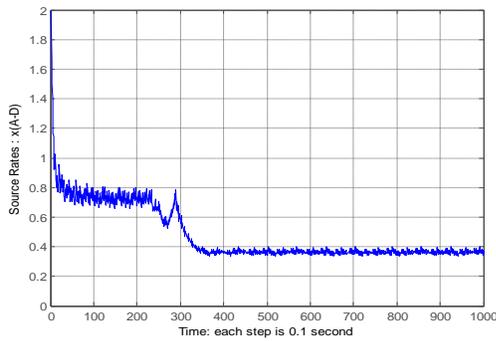


Figure 10. Source rate for flow  $A \rightarrow D$

If consider the source rate in Figure 10, for flow  $A \rightarrow D$ , we see the rate is decreased. Because this link have capacity of two, the average rate of it is higher than other links. Since the link lengths  $B \rightarrow D$  and  $A \rightarrow B$  are increased, the costs of them are increased, relatively. Therefore the source A decreases its rate.

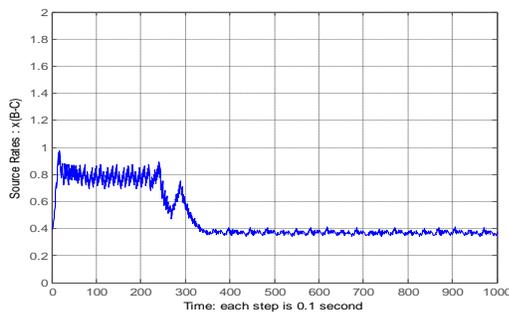


Figure 11. Source rate for flow  $B \rightarrow C$

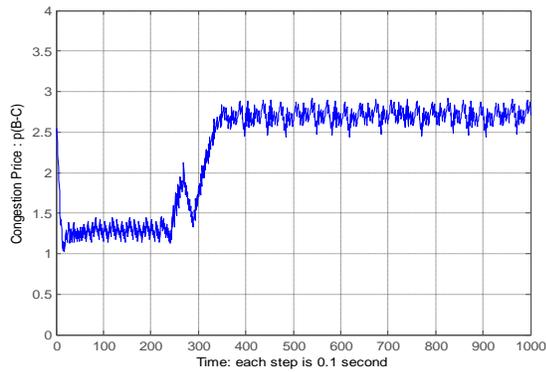


Figure 12. Congestion price for flow  $B \rightarrow C$

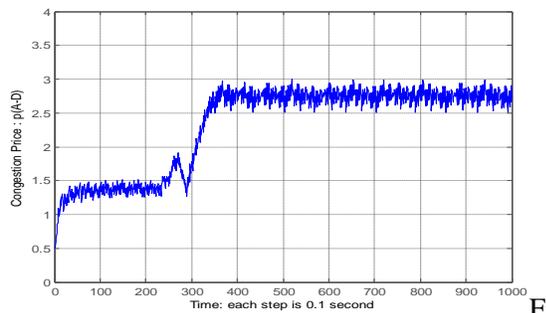


Figure 13. Congestion price for flow  $A \rightarrow D$

In Figures 14 and 15, we consider the average link rates for links  $C \rightarrow D$  and  $B \rightarrow D$ . The rate in link  $B \rightarrow D$  after mobility due to decrease of link length, is increased. The rate in link  $C \rightarrow D$  due to increase of link length, is decreased. In all figures of mobility section, we see the values, converge to stable value after 3-4 seconds. Thus if the time between distance change in mobility pattern be larger of this value, we can run our algorithm completely dynamically.

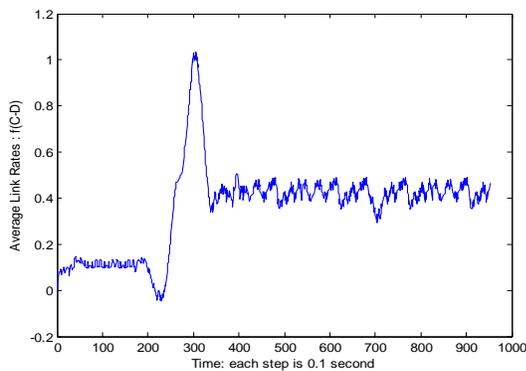
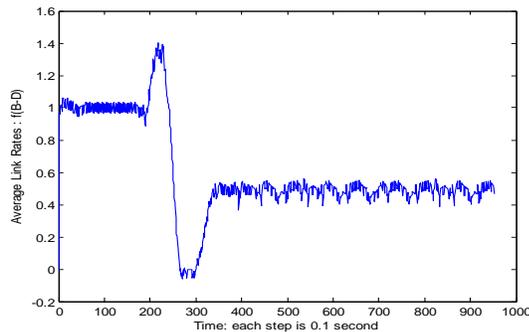


Figure 14. Average link rate for  $C \rightarrow D$

Figure 15. Average link rate for  $B \rightarrow D$ 

## 6. Discussions

We have presented a model for the mobility aware joint design of congestion control, routing and scheduling for ad hoc wireless networks by extending the framework of network utility maximization to utility-cost maximization and applying dual-based decompositions.

We formulate resource allocation in the network with fixed wireless channels or single-rate wireless devices as a utility maximization problem with schedulability and rate constraints arising from contention for the wireless channel. We add the mobility effect to utility problem by introducing link cost as a multi-commodity variable. By dual decomposition, we derive a sub gradient algorithm that is not only distributed spatially, but more interestingly, decomposes the system problem vertically into three protocol layers where congestion control, routing and scheduling jointly solve the network utility maximization problem.

Further research steps stemming out of this paper include the following. First, unique features in our algorithm for practical implementations need to be further leveraged. Second, we will extend the results to networks with more general interference models. Third, scheduling problem is always a challenging problem for ad hoc network, and continued exploration of distributed scheduling protocols will further enhance the performance gain from cross-layer design involving link layer. Fourth, we assume that the topology due to mobility is not changed. In next works, we must eliminate this assumption to have a more realistic model for network.

## References

1. Ren X, and Zhang J. Review of the Cross-Layer Design in Wireless Ad Hoc and Sensor Networks. 6th International Conference on Wireless Communications Networking and Mobile Computing (WiCOM), China, 2010.
2. Korger U, Hartmann C, Kusume K, Widmer J. Power control versus multi-user detection based cross-layer design in wireless adhoc networks. 21st International Symposium on Personal Indoor and Mobile Radio Communications (PIMRC). 2010.
3. Oh B. J, Chen Ch. W. A Cross-Layer Approach to Multichannel MAC Protocol Design for Video Streaming Over Wireless AdHoc Networks. IEEE Transactions on Multimedia. 2009; 11(6): 1052 – 1061.
4. Sanchez M, Manzoni P. A java-based ad hoc networks simulator. In Proceedings of the SCS Western Multiconference Web-based Simulation Track. 1999.
5. Eason G, Noble B, Sneddon I. N. On certain integrals of Lipschitz-Hankel type involving products of Bessel functions. Phil. Trans. Roy. Soc. London. 1955; vol. A247: 529–551.

6. Haas Z. a new routing protocol for reconfigurable wireless networks. In Proceedings of the IEEE International Conference on Universal Personal Communications (ICUPC). 1997; 562–565.
7. Camp T, Boleng J, Davies V. A survey of mobility models for ad hoc network research. *Wirel. Commun. Mob. Comput.* 2002; 2: 483–502 (DOI: 10.1002/wcm.72)
8. Chen L, Low S H, Doyle J. Cross-layer Congestion Control, Routing and Scheduling Design in Ad Hoc Wireless Networks. 2006.
9. Bertsekas D. *Nonlinear Programming*. 2nd ed. Athena scientific. 1999.
10. Chen L, Low S, Doyle J. Joint congestion control and media access control design for ad hoc wireless networks. *Proc. IEEE Infocom*. 2005.
11. Chiang M. Balancing transport and physical layers in wireless multihop networks: Jointly optimal congestion control and power control. *IEEE J. Sel. Area Comm.* 2005; 23(1): 104-116.
12. Chiang M, Low S. H, Calderbank R. A, Doyle J. C, Layering as optimization decomposition. *Proceedings of IEEE*. 2006.
13. Hajek B, Sasaki G. Link scheduling in polynomial time. *IEEE Trans. Information Theory*. 1988; 34: 910-917.
14. Hoepman J, Simple distribute weighted matchings, preprint, 2004; Available at <http://arxiv.org/abs/cs/0410047>.
15. Jain K, Padhye J, Padmanabhan V. N, Qiu L. Impact of interference on multi-hop wireless network performance. *Proc. ACM Mobicom*. 2003.
16. Kelly F. P, Maulloo A. K, Tan D. K. H. Rate control for communication networks: Shadow prices, proportional fairness and stability. *Journal of Operations Research Society*. 1998; 49(3):237-252.
17. Lee J. W, Chiang M, Calderbank R. A. Jointly optimal congestion and contention control in wireless ad hoc networks. *IEEE Communication Letters*. 2006.
18. Lin X, Shroff N. Joint rate control and scheduling in multihop wireless networks. 43th IEEE CDC. 2004.
19. Low S. H, Lapsley D. E. Optimal flow control, I: Basic algorithm and convergence. *IEEE/ACM Trans. on networking*. 1999; 7(6): 861-874.
20. Low S. H, A duality model of TCP and active queue management algorithms. *IEEE/ACM Transactions on Networking*. 2003.
21. Neely M, Modiano E, Rohrs C. Dynamic power allocation and routing for time varying wireless networks. *Proc. IEEE Infocom*, 2003. *Journal version*, *IEEE J. Sel. Area Comm.* 2005; 23(1): 89-103.
22. Papadimitriou C. H, Steiglitz K. *Combinatorial Optimization: Algorithm and Complexity*. Dover Publications. 1998.
23. Preis R. Linear time 1/2-approximation algorithm for maximum weighted matching in general graphs. 16th STACS, (Eds., C, Meinel and S. Tison), LNCS 1563, Springer. 1999.
24. Xiao L, Johnsson M, Boyd S. Simultaneous routing and resource allocation via dual decomposition. *IEEE Trans. Comm.* 2004.
25. Xue Y, Li B, Nahrstedt K. Price-based resource allocation in wireless ad hoc networks, *Proc. ACM IWQoS*. 2003.