Journal of Engineering Technology
Volume 6, Issue 1, Jan. 2018, PP. 1-12

RESEARCH ARTICLE -ENGINEERNG TECHNOLOGY

Following control of MIMO uncertain systems application to a water desalination system supplied by photovoltaic source

Abderrahmen Ben Chaabene, Khira Ouelhazi, Anis Sellami

Laboratory of Industrial Systems Engineering and Renewable Energies, Higher National School of Engineers of Tunis (ENSIT), 5 Avenue Taha Hussein, Tunisia

Abstract: All over the world the need for drinking and domestic water uses continues to increase indefinitely. To meet these needs several solutions are introduced such as reverse osmosis (RO) desalination. The major problem of this technique is its enormous electrical energy demand. Therefore, it is necessary to use renewable energy sources, in particular photovoltaic sources available in arid and semi-arid regions. Furthermore, the intermittence of the PV sources and its dependence with climatic conditions is also relevant. For these reasons, we have developed control techniques for efficient PV-ROD processes.
This paper presents two contributions. first one deals with a novel mathematic development of tracking control technique based on Variable Structure Model Reference Adaptive Control (VSMRAC) applied to uncertain MIMO linear systems. This technique control algorithm requires the system to follow a reference model by adjusting its dynamics and ensures minimal value of error between the plant dynamics and that of the reference model, even, in the case where the system presents parametric uncertainties in its model due to disturbances. The second presents a design of a coupling technology of water Reverse Osmosis Desalination (ROD) systems to photovoltaic (PV) sources to minimize the production cost.
Simulations and experimental results have been carried out using Matlab software and a real PV-RO water desalination prototype fed by a photovoltaic generator. These results show perfect performances of the control technique we developed to eliminate the effect of the parametric uncertainties of the model as well as the effect of the intermittence of the energy source.

Keywords: Desalination, Reverse Osmosis, Following Control, Photovoltaic, MIMO uncertain systems.

1. Introduction

As water is customarily used and needed in large quantities, it is important to minimize energy consumption whenever possible. In last decades, there were several initiatives to support the use of renewable or green energies in water domain [1,2]. Furthermore, process control is an important part of the desalination industry that requires to be operated at the optimum conditions. When coupled to PV sources [3], desalination technology becomes a complex process. Thus, the majority of dynamic models are of the form of uncertain Multi-Inputs, Multi-Outputs (MIMO) systems [4]. Driving these processes is a complex problem because of the interactions between the input parameters [5], the intermittence of solar energy and the effect of parametric uncertainties. In addition, in the case of an unstable electrical supply of the PV-ROD plants, parameters change their values with the time in a known range. Consequently, the design of control laws become a complex problem. Hence, our idea is to combine two types of robust controls: the adaptive control

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characterized by its real-time adjustment and the sliding mode control, characterized by its robustness [6]. Many formulae have been derived to tune the VSMRA controllers in the literature [7,8,9,10,11,12]. The use of this VSMRAC algorithm in the case of systems powered by photovoltaic sources presents a solution to avoid firstly, the effect of intermittence between the appearance and disappearance of the sun which causes the instability of the energy source and can destroy the PV-ROD system and, secondly minimizes the effect of disturbances.

Uncertainties affecting MIMO linear systems come from various sources [13]. They are due to following reasons: modelling errors, parametric variations, problems of precision for certain parameters, or approximations in dynamic models. Consequently, the representations of the uncertainties of multivariable systems differ according to their origin, and must be taken into account in the development of control laws. We have distinguished :

- Structured / unstructured uncertainties
- Parametric / nonparametric uncertainties
- Deterministic / stochastic uncertainties

In this study, we are interested in the parametric and deterministic structured uncertainties since in our PV-ROD system model we considered the effect of the input salinity of water as known perturbation.

This paper is organized as follows: The material and methods section is devoted to the theoretical elements explaining our mathematic development of the VSMRAC algorithms applied to MIMO linear uncertain system, to a background on PV sources and to the description of the PV-UV water disinfection system. The second section deals with the main simulated and experimental results then the most interesting conclusions.

2. Material and Methods

2.1. Theoretical elements

2.1.1. MIMO Linear uncertain systems

In this type of uncertainty, the vectors of the uncertain parameters on the dynamic matrices **A** and the control matrix **B** are measurable, the MIMO linear uncertain system can be represented in the state space by:

$$\dot{x} = (A + \Delta A)x + (B + \Delta B)u \tag{1}$$

Where ΔA and ΔB are respectively the uncertain components of the dynamic matrix **A** and the control matrix **B.**

By assumption, the pair (A, B) representing the nominal system is stabilizable and controllable. Uncertainties are then defined around the nominal system.

The generality of this model imposes that this type of uncertainty can be limited in standard, polytopic or verifying the hypothesis of matching conditions. The parametric uncertainties corresponding are bounded in norm.

The uncertainty matrices in the state representation of the system corresponds to equation (1) where:

$$\begin{bmatrix} \Delta A & \Delta B \end{bmatrix} = D_1 \nabla \begin{bmatrix} E_1 & E_2 \end{bmatrix} \tag{2}$$

$D_1 \in \Re^{nxd}$ $E_1 \in \Re^{exn}$ $E_2 \in \Re^{exm}$ are constant matrices characterizing the structure of uncertainty and $\nabla \in \Re^{dxe}$ is an uncertain matrix.

The uncertain components ΔA and ΔB are bounded in norm. These disturbances make it possible to take into account nonlinearities and neglected dynamics in the model of the system.

2.1.2. Novel Mathematic Development for the Following Control Strategy

In this approach, we will develop the mathematical formulations detailed in [14, 15, 16, 17] concerning the structure of the adaptive sliding mode control of multivariate systems by tracking a reference model, by replacing the multivariate system represented in the state space by equation (3).

$$\dot{x} = Ax + Bu \qquad (3)$$

by the uncertain linear multivariate system given by the equation (4).

$$\dot{x} = (A + \Delta A)x + (B + \Delta B)u \qquad (4)$$

whose uncertainties are structured parametric deterministic and bounded in norms.
Some assumptions are taken into account in this new approach:

Assumption1:
The reference trajectory in the state space is independent of the effect of uncertainties on the dynamics of the system.
Assumption2:
The trajectory of the system in the space $\mathbf{E_s}$ generates a cylindrical envelope whose axis is the reference trajectory $\mathbf{C_r}$ and whose radius r (ΔA) is depending on the effect of the uncertainties on the following error. This can be represented by the figure 1.

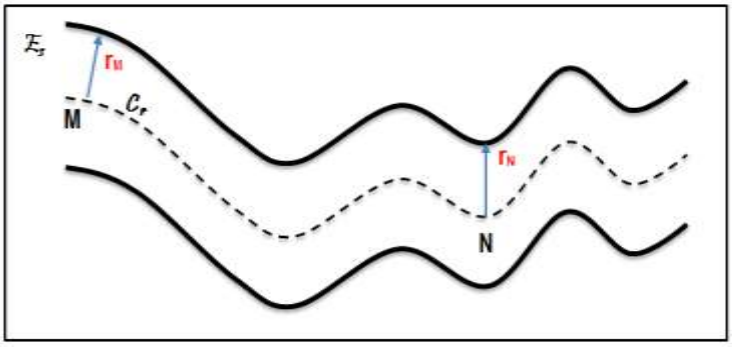

Figure 1. Effect of the uncertainties on the evolution of the continuation error

Consider the state representation of an uncertain system

$$\dot{x} = (A + \Delta A)x + Bu \qquad (5)$$

The reference model is represented by the equation

$$\dot{x}_r = A_r x_r + B_r u_r \qquad (6)$$

To determine the matrices of the state vector of the reference model A_r and B_r, two matrices Θ^* and Q^* are defined such that

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$$A_r = A + \Delta A + B\theta^* \quad (7)$$

$$B_r = BQ^* \quad (8)$$

With :
Θ^* : matrix with dimension (m x n)
Q^* : diagonal matrix (m x m)
The control law stretching the error to zero is defined

$$u = \psi x + Q^* u_r \quad (9)$$

Where Ψ is a counter-reaction matrix of dimension (m x n). The elements Ψ_{ij} of the matrix Ψ are switching functions adjusted by a variable structure approach.The tracking error for the uncertain system:

$$x_{ei} = x - x_r \quad (10)$$

Or

$$\dot{x}_{ei} = \dot{x}_i - \dot{x}_{rm} \quad (11)$$

By replacing $\dot{x}$ and $\dot{x}_r$ by their expressions in (5) and (6), we obtain:

$$\dot{x}_{ei} = (A + \Delta A)x + Bu - (A_r x_r + B_r u_r) \quad (12)$$

$$A = A_r - \Delta A - B\theta^* \quad (13)$$

$$\dot{x}_{ei} = (A_r - B\theta^*)x + Bu - A_r X_r - B_r u_r \quad (14)$$

By replacing the control law with its expression in (9), and using the expression (14) we obtain:

$$\dot{x}_{ei} = A_r(x - x_r) + B(\psi - \theta^*)x \quad (15)$$

(

By comparing the expression (15), by expressions in literature such as detailed in [14] for the certain systems (or systems without uncertainties), we find that the derivative of the state error for the uncertain system has the same expression than (16) as follows

$$\frac{de}{dt} = A_r e + B\left(\psi - \Omega^*\right)X \quad (16)$$

so we can also observe two performances :
- A judicious choice of the matrices **F** and **H** makes it possible to eliminate the effect of the uncertainties on the system dynamics .
- For Lyapunov-based stability with the function

$$\Lambda = e^T P e \quad (17)$$

The derivative eliminates the constant parametric uncertainties on the matrices **A** and **B**. Consequently, the stability of the system is not affected.
- Following error depends on the uncertainty on the system state matrix.
These findings are demonstrated by simulations and experimental results in paragraph (3).

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2.2. PV-RO water desalination system

2.2.1. PV-RO system Design

The ROD system design, shsown in Figure 2 is essentially composed of:

- A Photovoltaic generator
- High Pressure (HP) pump
- A reverse osmosis unit

The RO unit is the principal component of the ROD system. It is a semi-permeable polyamide membrane, composed of two sides: a brine side and a permeate side. The brackish or sea-water is pumped into a closed vessel and pressurized by a High Pressure (HP) pump against the membrane where the salt solution is rejected by the brine side, but the desalted pure water passes through the permeate side.

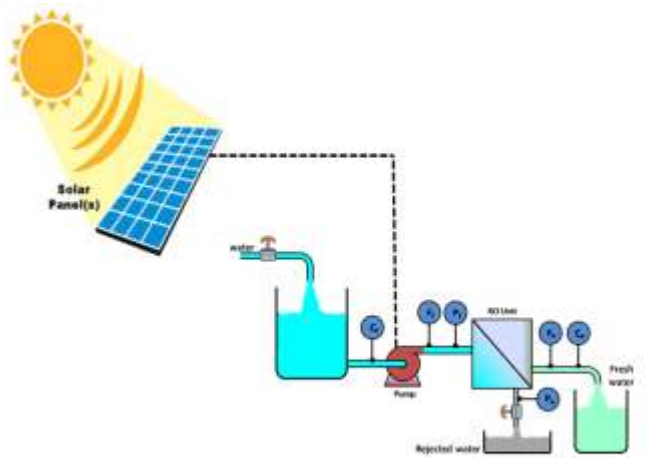


Figure 2. Design of the PV- RO water desalination system

2.2.2. The PV generator

B. Chaabene et al. [18] detailed, the principle of the use of photovoltaic generators. They consist of mixed association of photovoltaic cells to provide the desired DC voltage and current. The coupling of the battery with the PV generator is realized through a charge regulator, the electrical model of the PV generator is shown in Figure 3.

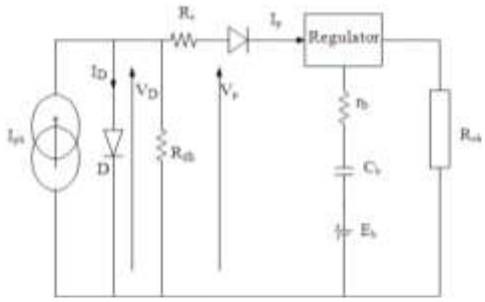


Figure 3. The energetic system equivalent diagram

The electrical model of the PV generator is given by the following expressions detailed in [18].

$$V_D = V_P + R_S I_P$$

(18)

$$I_P = I_{ph} - I_D - (V_D / R_{sh})$$

$$I_P = I_{ph} - I_s[\exp(qV_D/AKT) - 1] - V_D/R_{sh}$$

I_P : Photo current (A)

I_S : Reverse saturation current (A)

Q : Elementary charge (C)

V_P : Terminal voltage (V)

V_D : Diode voltage (V)

R_{SH} : Shunt resistance caused by the cell surface state (Ω)

R_S Serial resistance caused by the based resistance of the junction face (Ω)

K : Boltzmann constant = 1.38064852(79)×10−23 J/K.T

T: Absolute temperature (K)

2.2.3. Experimental plant

Figure 3 shows the set-up of the RO desalination experimental platform installed in the Research and Technology Center of Energy at the Technology Park of Borj-Cedria, Hammam-Lif, Tunisia. As it is shown, the ROD system is composed of a feed water pump, three reverse osmosis modules containing the membranes and a data acquisition system equipped with sensors to measure the flow rate, the salinity and the pressure.

Figure3. RO Water Desalination Experimental set-up

3. Simulation and Experimental Results

3.1. ROD state space model

Using Matlab software, we have simulate the ROD system dynamic. The state space model of the system given by expression (5) was developed in [18] and represented by the following constant matrices A, B and C determined from experimental results

$$A=\begin{pmatrix}-1 & 1 & 0 & 0\\ -2.25 & -1.50 & 0 & 0\\ 0 & 0 & -1 & 1\\ 0 & 0 & -4.62 & -3.23\end{pmatrix} B=\begin{pmatrix}2.50 & 0\\ 0 & -0.56\\ 0 & -0.20\\ -0.81 & 0\end{pmatrix} C=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{pmatrix} \Delta A=\begin{pmatrix}0.09 & 0 & 0 & 0\\ 0.81 & 0.78 & 0 & 0\\ 0 & 0 & 0.09 & 0\\ 0 & 0 & 1.66 & 1.68\end{pmatrix}$$

3.2. VSMRA Control algorithm

Figure 4 shows the algorithm for calculating the VSMRA control of the PV-RO desalination system. The chronology of the calculation steps of the control first requires the calculation of the switching function and the control law to decrease the following error, then forces the system to follow its reference model.

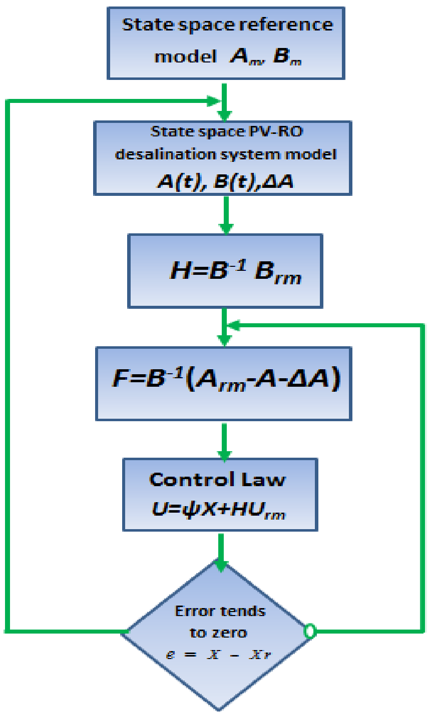


Figure 4. Algorithm for calculating the VSMRA control of the PV-RO desalination system

Figures 5 shows the Matlab Simulink model of the PV-RO desalination system corresponding to the described algorithm.

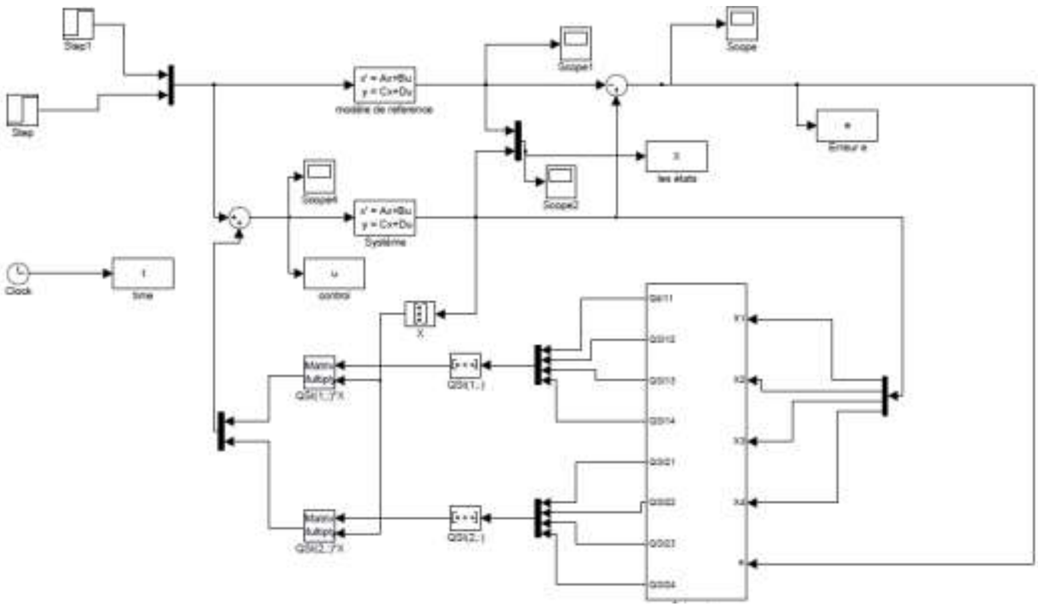


Figure 6. Mathlab Simulink model of the PV-UV following control

3.3. Solar irradiation

As we have determined in [17] in order to know the solar deposit, Fig.7. shows the variation of solar irradiation. We can deduce that the available irradiance measured over two days test is with important quantities (more than 500 W.m^{-2} while at least 8 hours a day).

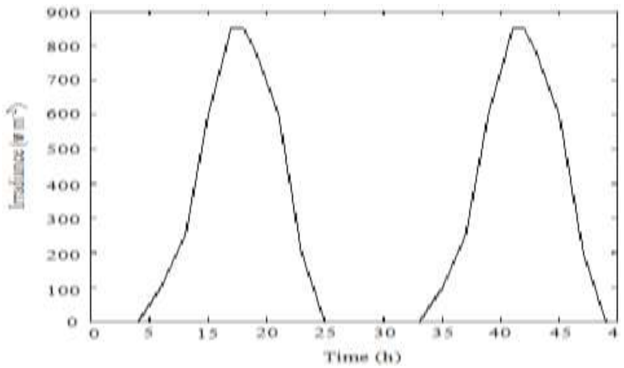
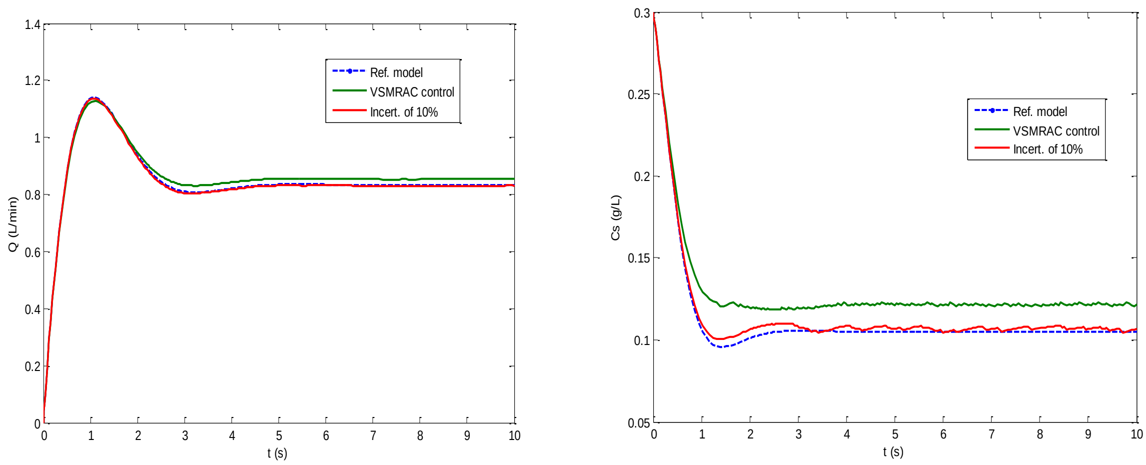


Figure 7. The Measured solar Irradiance PV-UV water disinfection dynamic

3.4.Test of Tracking dynamics with different uncertainties

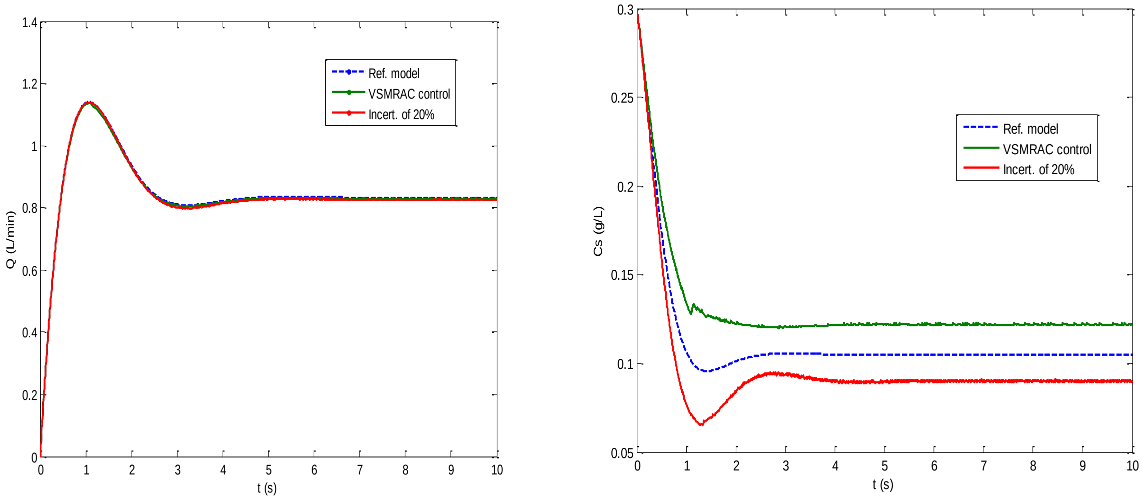
In order to test the dynamics of PV-ROD system, taking into account the parametric uncertainties in its model, we proceed to impose the reference model as well as the uncertainty and to see the results of simulations as well as the real evolution of the dynamics of the system. the following curves show the tracking performance for parametric uncertainties of different values 10%, 20% and 40%. The choice of these values of uncertainties stems from the fact that a preliminary study shows that they do not often exceed 25%.



(a) **(b)**

Figure 8. Tracking dynamics for an uncertainty of 10% (a) Flow rate, (b)Salinity

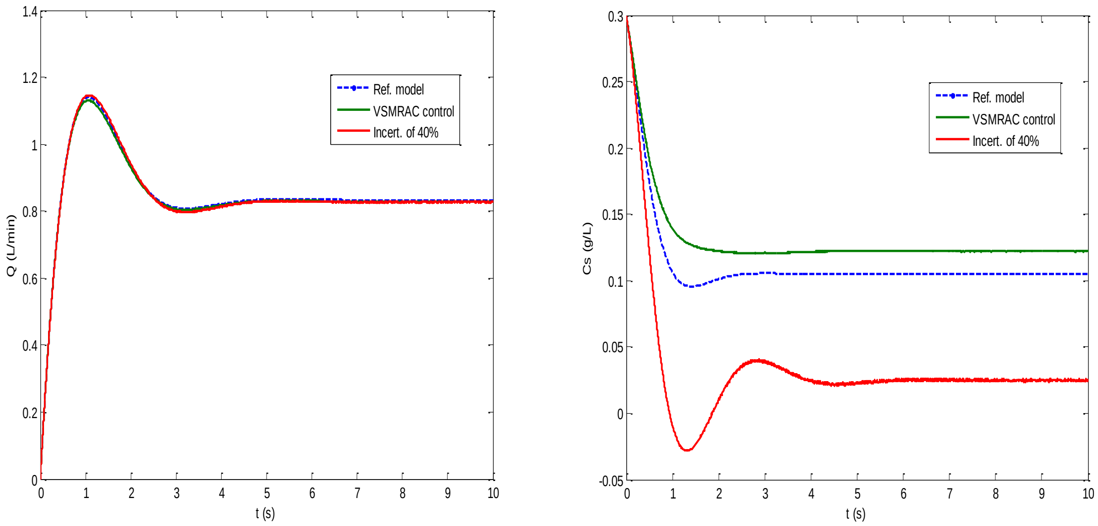
Figures 8 (a) and 8 (b) show respectively, the step responses of the product water flow rate **Q** and the product salinity **Cs** for an uncertainty of 10%, the reference trajectory is characterized by a slow dynamics. Curves show perfect model following at a finite time. The tracking error values do not exceed 3% for both of the two outputs **Q** and **Cs**. This is the consequence of the performances combination of the real time adjustment of the adaptive control with the robustness of the variable structure control. In addition, the two output parameters converge to fixed values, hence the stability of the system even in the presence of uncertainties.



(a) **(b)**

Figure 9. Tracking dynamics for an uncertainty of 20% : (a) Flow rate, (b)Salinity

Figures 9 (a) and 9 (b) show respectively the step responses of the product water flow rate **Q** and the product salinity **Cs** for an uncertainty of 20%. Curves show perfect model following at a finite time for the flow rate Q . For the salinity Cs, the tracking error values reaches 5%. This is considered to be low in relation to the significant uncertainty. The stability of the system is kept even for this uncertainty value.



(a) **(b)**

Figure10. Tracking dynamics for an uncertainty of 40% : (a) Flow rate, (b)Salinity

Figures 10 (a) and 10 (b) show respectively the step responses of the product water flow rate **Q** and the product salinity **Cs** for an uncertainty of 40%. Even for this very high level of uncertainty, curves show perfect model following at a finite time for the flow rate Q and tracking error values not exceeding 8% for the salinity Cs. The tracking is deemed to be perfect in relation to the high degree of uncertainty. The stability of the system is always kept even for this uncertainty value.

3.5. Performances of the proposed Control

From previous figures, it can be seen that :

-The tracking error slightly increases with the uncertainties values but remains clearly low, compared to these values.

-whatever the values of the uncertainties are, the system stability is always kept.

Therefore it can be concluded that due to combination of two robust controls, the VSMRA Control is suitable for uncertain linear multivariate systems, it keeps the stability of the system and eliminates the effect of uncertainties on dynamics behavior. Simulations and experimental results prove our theoretical results presented at the beginning of this paper which showed that the uncertainty is not included in the expression (15) of the derivation of the tracking error.

The comparison of the results mentioned by the curves for the two output parameters (water flow rate and salinity) of the ROD system shows that the effect of the uncertainties on the dynamic behavior has been eliminated since a low value of the error of the system has been kept (not exceeding 8%). In addition the intermittence of the energy source is neglected by the type of control which requires the system to follow its reference model independently of the external factors. Moreover these results justify the theoretical ones which show that the effect of the uncertainties is mathematically eliminated, the ROD system behavior of is then identical to that of a non-perturbed system.

4.Conclusion

In this paper, a VSMRA Control for MIMO uncertain linear systems has been proposed which assures the system stability. A mathematical development has been presented which shows the suppression of the uncertainties effects on the tracking stability and the following error value. Furthermore, the problem of variation of the PV-ROD system dynamics due to the intermittency of the PV source was also solved by the tracking of the reference model. To validate the theoretical study, simulation and experimental results have been presented to illustrate the effectiveness of the proposed control technique.

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