Journal of Engineering Technology
Volume 6, Issue 1, Jan. 2018, PP. 13-27

RESEARCH ARTICLE -ENGINEERNG TECHNOLOGY

F-root square mean labeling of graphs obtained from some graph operations

S. Arockiaraj[1], A. Durai Baskar[2], A. Rajesh Kannan[2]

[1]Department of Mathematics, Government Arts & Science College, Sivakasi- 626124, Tamilnadu, India.

[2]Department of Mathematics, Mepco Schlenk Engineering College, Sivakasi- 626005, Tamilnadu, India.

Abstract: A function f is called a F-root square mean labeling of a graph $G(V,E)$ with p vertices and q edges if $f: V(G) \rightarrow \{1,2,3,\ldots,q+1\}$ is injective and the induced function f^* defined as $f^*(uv) = \left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$ for all $uv \in E(G)$, is bijective. A graph that admits a F-root square mean labeling is called a F-root square mean graph. In this paper, we have discussed the F-root square meanness of the planar grid $P_m \times P_n$, for $m \leq 5$, the graph $L_n \circ S_m$, for $3 \leq m \leq 4$, the graph P_a^b for $a \geq 2$ and $b \leq 3$, the graph D_n^*, the latitude graph, the graph $P_n(X_1, X_2, \ldots, X_n)$, the splitting graph $S'(P_n)$ and the arbitrary supersubdivision graph $P(m_1, m_2, \ldots, m_{n-1})$ of the path P_n.

Keywords: Labeling, F-root square mean labeling, F-root square mean graph.

2010 Mathematics Subject Classification Number: 05C78.

1. Introduction

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $G(V,E)$ be a graph with p vertices and q edges. For notations and terminology, we follow [10]. For a detailed survey on graph labeling, we refer [9].

A path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n. A star graph S_n is the complete bipartite graph $K_{1,n}$. The graph $G \circ S_m$ is obtained from G by attaching m pendant vertices to each vertex of G.

Let G_1 and G_2 be any two graphs with p_1 and p_2 vertices, respectively. Then the cartesian product $G_1 \times G_2$ has $p_1 p_2$ vertices with vertex set $\{(u,v): u \in G_1, v \in G_2\}$ and the edges are obtained as follows: (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \times G_2$ if either $u_1 = u_2$ and v_1 and v_2 are adjacent in G_2 or u_1 and u_2 are adjacent in G_1 and $v_1 = v_2$. The product $P_m \times P_n$ and is called a planar grid and $P_2 \times P_n$ is called a ladder, denoted by L_n. Let a and b be integers such that $a \geq 2$ and $b \geq 2$. Let $y_1, y_2, \ldots, y_a$ be the 'a' fixed vertices. Connect y_i and y_{i+1} by means of b internally disjoint paths P_i^j of length '$i+1$' each, for $1 \leq i \leq a-1$ and $1 \leq j \leq b$. The resulting graph embedded in the plane is denoted by P_a^b. The graph D_n^* having the vertices $\{a_{i,j} : 1 \le i \le n,\ j = 1,2,3,4\}$ and its edge set is $\{a_{i,1}\,a_{i+1,1}, a_{i,3}\,a_{i+1,3} : 1 \le i \le n-1\} \cup \{a_{i,1}\,a_{i,2}, a_{i,2}\,a_{i,3}, a_{i,3}\,a_{i,4}, a_{i,4}\,a_{i,1} : 1 \le i \le n\}$. The latitude graph is a graph obtained from a cycle $u_1u_2\ldots,u_nu_1$ of even length n by adjoining u_i and u_{n+2-i}, for $2 \le i \le n/2$.

The graph $P_n(X_1, X_2,\ldots,X_n)$ is a tree obtained from a path on n vertices by attaching X_i pendent vertices at each ith vertex of the path, for $1 \le i \le n$. Let G be a graph. For each vertex v of G, take a new vertex v' to these vertices of G adjacent to v. The graph thus obtained is called the splitting graph G and it is denoted by $S'(G)$. An arbitrary super subdivision $P(m_1, m_2,\ldots,m_{n-1})$ of the path P_n, is a graph obtained by replacing each ith edge of P_n, by identifying its end vertices of the edge with a partition of K_{2,m_i} having two elements, where m_i is any positive integer.

Vasuki et al. discussed the even vertex odd mean labeling of the planar grid $P_m \times P_n$, for $m \ge 2$ and $n \ge 2$ [17]. Arockiaraj et al. studied the F- root square meanness of the graph $L_n \circ S_m$, for $n \ge 2$ and $m \le 2$ [3]. Vasuki and Nagarajan proved that the graph P_r^{2m+1} is an odd mean graph for all values of r and n [16]. Mathew Varkey and Sunoj showed that the graph D_n^* is odd graceful, for any n [13]. Meenakumari and Arockiaraj studied that $(1,1,0)$ F-Face magic meanness of the latitude graph [14]. Durai Baskar et al. investigated the F-Geometric meanness of the graph labeling of the $P_n(X_1, X_2,\ldots,X_n)$ [5]. Lawrence Rozario Raj and Koilraj proved that the splitting graph of the path is cordial [12]. Karthiresan et al. showed that the arbitrary supersubdivision of any star is graceful [11]. Motivated by these works, the F-root square meanness property of the planar grid $P_m \times P_n$ for $m \le 5$, the graph $L_n \circ S_m$ for $3 \le m \le 4$, the graph P_a^b for $a \ge 2$ and $b \le 3$, the graph D_n^*, the latitude graph, the graph $P_n(X_1, X_2,\ldots,X_n)$, the splitting graph $S'(P_n)$ and the arbitrary supersubdivision graph $P(m_1, m_2,\ldots,m_{n-1})$ of the path P_n have been discussed.

The concept of F-Geometric mean labeling in [7] was introduced by Durai Baskar et al. and studied some standard graphs in [6], [8]. The concept of Root square mean labeling was introduced and studied by Sandhya et al. [15]. Motivated by the works of so many authors in the area of graph labeling, we introduced a new type of labeling called F-root square mean labeling in [1] and the F- root square meanness property for some standard graphs was studied in [2], [3], [4].

In [15], the root square mean labeling is defined as follows: A graph $G(V, E)$ with p vertices and q edges is said to be Root square mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1,2,3,\cdots,q+1$ in such a way that when each edge $e = uv$ is labeled with either $\left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$, then the edge labels are distinct. In this case, f is called a root square mean labeling of G. In the above definition, the readers will get some confusion in finding the edge labels which edge is assigned by flooring function and which edge is assigned by ceiling function. To avoid the confusion of assigning the edge labels in their definition, we just consider the flooring function $\left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$ for our discussion.

A function f is called a F-root square mean labeling of a graph $G(V,E)$ with p vertices and q edges if $f:V(G)\to\{1,2,3,\ldots,q+1\}$ is injective and the induced function f^* defined as $f^*(uv) = \left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$ for all $uv \in E(G)$, is bijective. A graph that admits a F-root square mean labeling is called a F-root square mean graph [1].

Arockiaraj et al. [1] have studied the F-root square meanness of the path P_n, the graph $P_n \circ S_m$, the graph $P_n \circ K_2$, the graph $TW(P_n)$, the graph $[P_n;S_m]$, the graph $S(P_n \circ K_1)$, the graph $M(P_n)$, the graph $T(P_n)$, the graph P_n^2, the ladder graph L_n and the slanting ladder graph SL_n. Arockiaraj et al. [2] have analyzed the F-root square meanness of the graph obtained from a path by replacing any of its edges by a cycle, the cycle graph C_n, the graph obtained by identifying a vertex of any two cycles, the triangular snake T_n, the alternate triangular snake AT_n, the quadrilateral snake Q_n, the alternate quadrilateral snake AQ_n, the tadpoles $T(n,k)$, the graph $C_n \circ K_1$ and the triangular ladder graph TL_n.

Arockiaraj et al. [3] have proved the F-root square meanness of graphs $L_n \circ S_m$ for $m \le 2$, $TL_n \circ S_m$ for $m \le 2$, $SL_n \circ S_m$ for $m \le 2$, double sided step ladder graph $2ST_{2n}$ and one sided step ladder graph ST_n. Arockiaraj et al. [4] have discussed the F-root square mean labeling of various graphs resulted from the duplication of graph elements.

In this paper, we have discussed the F-root square meanness of the planar grid $P_m \times P_n$, for $m \le 5$, the graph $L_n \circ S_m$, for $3 \le m \le 4$, the graph P_a^b for $a \ge 2$ and $b \le 3$, the graph D_n^*, the latitude graph, the graph $P_n(X_1, X_2, \ldots, X_n)$, the splitting graph $S'(P_n)$ and the arbitrary supersubdivision graph $P(m_1, m_2, \ldots, m_{n-1})$ of the path P_n.

Theorem 1.1 [3] The graph $L_n \circ S_m$ is a F- root square mean graph, for $n \ge 2$ and $m \le 2$.

2. F- root square Meanness of some planar graphs

Theorem 2.1. The planar grid $P_m \times P_n$ is a F- root square mean graph, for $m \le 5$ and $n \ge 2$.
Proof. Let $V(P_m \times P_n) = \{v_{ij} : 1 \le i \le m, 1 \le j \le n\}$ and $E(P_m \times P_n) = \{v_{ij}v_{(i+1)j} : 1 \le i \le m-1, 1 \le j \le n\} \cup \{v_{ij}v_{i(j+1)} : 1 \le i \le m, 1 \le j \le n-1\}$ be the vertex set and edge set of the graph $P_m \times P_n$.
Case (i). $m = 2$.
A vertex labeling $f : V(P_2 \times P_n) \to \{1,2,3,\ldots,3n-1\}$ is defined as

Journal of Engineering Technology
Volume 6, Issue 1, Jan. 2018, PP. 13-27
RESEARCH ARTICLE -ENGINEERNG TECHNOLOGY

$f(v_{ij}) = i + 3(j-1)$, for $1 \le i \le 2$ and $1 \le j \le n$.

Then the induced edge labeling f^* is obtained as follows.

$f^*(v_{1j}v_{2j}) = 3j - 2$, for $1 \le i \le n$ and $f^*(v_{ij}v_{i(j+1)}) = i + 3j - 2$, for $1 \le i \le 2$ and $1 \le j \le n-1$.

Case (ii). $m = 3$.

Define $f : V(P_3 \times P_n) \to \{1,2,3,\ldots,5n-2\}$ as follows.

$f(v_{i1}) = i$, for $1 \le i \le 3$,

$$f(v_{i2}) = \begin{cases} i+4, & 1 \le i \le 2 \\ 8, & i = 3 \quad and \end{cases}$$

$f(v_{ij}) = i + 5(j-1)$, for $1 \le i \le 3$ and $3 \le j \le n$.

Then the induced edge labeling f^* is obtained as follows.

$f^*(v_{i1}v_{(i+1)1}) = i$, for $1 \le i \le 2$,

$$f^*(v_{i1}v_{i2}) = \begin{cases} i+2, & 1 \le i \le 2 \\ 6, & i = 3 \end{cases}$$

$$f^*(v_{ij}v_{(i+1)j}) = \begin{cases} 2i + 3(j-1), & 1 \le i \le 2 \ and \ j = 2 \\ i + 5(j-1), & 1 \le i \le 2 \ and \ 3 \le j \le n \ and \end{cases}$$

$f^*(v_{ij}v_{i(j+1)}) = i + 5j - 3$, for $1 \le i \le 3$ and $2 \le j \le n-1$.

Case (iii). $m = 4$.

Define $f : V(P_4 \times P_n) \to \{1,2,3,\ldots,7n-3\}$ as follows.

$f(v_{i1}) = i$, for $1 \le i \le 4$, $\quad f(v_{i2}) = \begin{cases} i+5, & 1 \le i \le 2 \\ 14-i, & 3 \le i \le 4 \quad and \end{cases}$

$$f(v_{ij}) = \begin{cases} i+7j-6, & i=1 \ and \ 3 \le j \le n \\ i+7j-9, & i=2 \ and \ 3 \le j \le n \ and \ j \ is \ odd \\ i+7j-8, & i=2 \ and \ 3 \le j \le n \ and \ j \ is \ even \\ i+7j-6, & i=3 \ and \ 3 \le j \le n \\ i+7j-8, & i=4 \ and \ 3 \le j \le n. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows.

$f^*(v_{i1}v_{(i+1)1}) = i$, for $1 \le i \le 3$,

$$f^*(v_{i2}v_{(i+1)2}) = \begin{cases} 6, & i = 1 \\ i+7, & 2 \le i \le 3, \end{cases}$$

Journal of Engineering Technology
Volume 6, Issue 1, Jan. 2018, PP. 13-27

$$f^*(v_{i1}v_{i2}) = \begin{cases} 3i-1, & 2 \le i \le 3 \\ 7, & i = 4, \end{cases}$$

$$f^*(v_{1j}v_{1(j+1)}) = \begin{cases} 8j-4, & 1 \le j \le 2 \\ 7j-2, & 3 \le j \le n-1, \end{cases}$$

$$f^*(v_{ij}v_{i(j+1)}) = \begin{cases} 7j, & i = 3 \text{ and } 2 \le j \le n-1 \\ i+7j-5, & i = 2,4 \text{ and } 2 \le j \le n-1 \text{ and} \end{cases}$$

$$f^*(v_{ij}v_{(i+1)j}) = i + 7(j-1), \text{ for } 1 \le i \le 3 \text{ and } 3 \le j \le n.$$

Case (iv). $m = 5$.
Define $f : V(P_5 \times P_n) \to \{1,2,3,\ldots,9n-4\}$ as follows.
$f(v_{i1}) = i$, for $1 \le i \le 5$,

$$f(v_{i2}) = \begin{cases} i+11, & 1 \le i \le 3 \\ i+3, & 4 \le i \le 5 \text{ and} \end{cases}$$

$$f(v_{ij}) = \begin{cases} i+9j-6, & 1 \le i \le 2 \text{ and } 3 \le j \le n \\ i+9j-11, & 3 \le i \le 5 \text{ and } 3 \le j \le n. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows.

$$f^*(v_{1j}v_{1(j+1)}) = 9j-1, \text{ for } 1 \le j \le n-1,$$

$$f^*(v_{2j}v_{2(j+1)}) = 9j, \text{ for } 1 \le j \le n-1,$$

$$f^*(v_{3j}v_{3(j+1)}) = 6j+4, \text{ for } 1 \le j \le 2,$$

$$f^*(v_{i1}v_{(i+1)1}) = i, \text{ for } 1 \le i \le n-1,$$

$$f^*(v_{ij}v_{i(j+1)}) = \begin{cases} i+9j-8, & 4 \le i \le 5 \text{ and } 1 \le j \le 2 \\ i+9j-7, & 4 \le i \le 5 \text{ and } 3 \le j \le n-1, \end{cases}$$

$$f^*(v_{i2}v_{(i+1)2}) = \begin{cases} i+11, & 1 \le i \le 2 \\ 11, & i = 3 \\ 7, & i = 4 \text{ and} \end{cases}$$

$$f^*(v_{ij}v_{(i+1)j}) = \begin{cases} i+9j-6, & i = 1 \text{ and } 3 \le j \le n \\ i+9j-8, & i = 2 \text{ and } 3 \le j \le n \\ i+7j-11, & 3 \le i \le 4 \text{ and } 3 \le j \le n. \end{cases}$$

Hence f is a F- root square mean labeling of $P_m \times P_n$, for $m \le 5$ and $n \ge 2$. Thus the graph $P_m \times P_n$ is a F- root square mean graph, for $m \le 5$ and $n \ge 2$.

Theorem 2.2. The graph $L_n \circ S_m$ is a F- root square mean graph, for $n \geq 2$ and $3 \leq m \leq 4$.

Proof. Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be the vertices of the ladder L_n. Let $u_1^{(i)}, u_2^{(i)}, u_3^{(i)}, \ldots, u_m^{(i)}$ and $v_1^{(i)}, v_2^{(i)}, v_3^{(i)}, \ldots, v_m^{(i)}$ be the pendant vertices at each u_i and v_i respectively, for $1 \leq i \leq n$. Then the edge set is $E(L_n \circ S_m) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq n\} \cup \left\{u_i u_j^{(i)} : 1 \leq i \leq n, 1 \leq j \leq m\right\} \cup$
$\left\{v_i v_j^{(i)} : 1 \leq i \leq n, 1 \leq j \leq m\right\}$.

Case (i). $m = 3$.

Define $f : V(L_n \circ S_3) \to \{1, 2, 3, \ldots, 9n-1\}$ as follows.

$$f(u_i) = \begin{cases} 2, & i = 1 \\ 9i-8, & 2 \leq i \leq n \text{ and } i \text{ is odd} \\ 9i-2, & 2 \leq i \leq n \text{ and } i \text{ is even,} \end{cases}$$

$$f(v_i) = \begin{cases} 6, & i = 1 \\ 9i-2, & 2 \leq i \leq n \text{ and } i \text{ is odd} \\ 9i-8, & 2 \leq i \leq 6 \text{ and } i \text{ is even} \\ 9i-7, & 8 \leq i \leq n \text{ and } i \text{ is even,} \end{cases}$$

$$f\left(v_1^{(i)}\right) = \begin{cases} 9i-5, & 1 \leq i \leq n \text{ and } i \text{ is odd.} \\ 9, & i = 2 \\ 9i-7, & 4 \leq i \leq 6 \text{ and } i \text{ is even} \\ 9i-8, & 8 \leq i \leq n \text{ and } i \text{ is even,} \end{cases} \qquad f\left(u_1^{(i)}\right) = \begin{cases} 1, & i = 1 \\ 18, & i = 3 \\ 9i-7, & 4 \leq i \leq n \text{ and } i \text{ is odd} \\ 9i-5, & 2 \leq i \leq n \text{ and } i \text{ is even,} \end{cases}$$

$$f\left(u_2^{(i)}\right) = \begin{cases} 9i-6, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 9i-3 & 1 \leq i \leq n \text{ and } i \text{ is even,} \end{cases} \qquad f\left(v_2^{(i)}\right) = \begin{cases} 4i+3, & 1 \leq i \leq 2 \\ 9i-3, & 3 \leq i \leq n \text{ and } i \text{ is odd} \\ 9i-6, & 3 \leq i \leq n \text{ and } i \text{ is even and} \end{cases}$$

$$f\left(u_3^{(i)}\right) = \begin{cases} 9i-4, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 9i-1, & 1 \leq i \leq n \text{ and } i \text{ is even,} \end{cases}$$

$$f\left(v_3^{(i)}\right) = \begin{cases} 9i-1, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 9i-4, & 1 \leq i \leq n \text{ and } i \text{ is even.} \end{cases}$$

Then the induced edge labeling f^* is obtained as follows.

$$f^*(u_i u_{i+1}) = \begin{cases} 11, & i = 1 \\ 9i, & 3 \leq i \leq 5 \text{ and } i \text{ is odd} \\ 9i-1, & 7 \leq i \leq n-1 \text{ and } i \text{ is odd} \\ 9i-1, & 2 \leq i \leq n-1 \text{ and } i \text{ is even,} \end{cases} \qquad f^*(v_i v_{i+1}) = \begin{cases} 9i-1, & 1 \leq i \leq 5 \text{ and } i \text{ is odd} \\ 9i, & 6 \leq i \leq n-1 \text{ and } i \text{ is odd} \\ 19, & i = 2 \\ 9i, & 4 \leq i \leq n-1 \text{ and } i \text{ is even,} \end{cases}$$

$$f^*(u_i v_i) = 9i-5, \text{ for } 1 \leq i \leq n,$$

Journal of Engineering Technology
Volume 6, Issue 1, Jan. 2018, PP. 13-27

$$f^*\left(u_i u_1^{(i)}\right) = \begin{cases} 1, & i=1 \\ 18, & i=3 \\ 9i-8, & 4 \le i \le n \text{ and } i \text{ is odd} \\ 9i-4, & 1 \le i \le n \text{ and } i \text{ is even,} \end{cases}$$

$$f^*\left(u_i u_2^{(i)}\right) = \begin{cases} 9i-7, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 9i-3, & 1 \le i \le n \text{ and } i \text{ is even,} \end{cases} \quad f^*\left(u_i u_3^{(i)}\right) = \begin{cases} 9i-6, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 9i-2, & 1 \le i \le n \text{ and } i \text{ is even,} \end{cases}$$

$$f^*\left(v_i v_1^{(i)}\right) = \begin{cases} 9i-4, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 9, & i=2 \\ 9i-8, & 4 \le i \le n \text{ and } i \text{ is even,} \end{cases} \quad f^*\left(v_i v_2^{(i)}\right) = \begin{cases} 9i-3, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 10, & i=2 \\ 9i-7, & 4 \le i \le n \text{ and } i \text{ is even and} \end{cases}$$

$$f^*\left(v_i v_3^{(i)}\right) = \begin{cases} 9i-2, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 9i-6, & 1 \le i \le n \text{ and } i \text{ is even.} \end{cases}$$

Case (ii). $m = 4$.

Define $f : V(L_n \circ S_4) \to \{1,2,3,\ldots,11n-1\}$ as follows.

$$f(u_i) = \begin{cases} 2, & i=1 \\ 11i-10, & 2 \le i \le n \text{ and } i \text{ is odd} \\ 11i-2, & 2 \le i \le n \text{ and } i \text{ is even,} \end{cases}$$

$$f(v_i) = \begin{cases} 8, & i=1 \\ 11i-2, & 2 \le i \le n \text{ and } i \text{ is odd} \\ 11i-10, & 2 \le i \le n \text{ and } i \text{ is even,} \end{cases}$$

$$f\left(u_1^{(i)}\right) = \begin{cases} 1, & i=1 \\ 7i+1, & 2 \le i \le 3 \\ 11i-9, & 4 \le i \le n \text{ and } i \text{ is odd} \\ 11i-8, & 4 \le i \le n \text{ and } i \text{ is even,} \end{cases} \quad f\left(u_2^{(i)}\right) = \begin{cases} 14i-11, & 1 \le i \le 2 \\ 11i-8, & 3 \le i \le n \text{ and } i \text{ is odd} \\ 11i-6, & 3 \le i \le n \text{ and } i \text{ is even,} \end{cases}$$

$$f\left(u_3^{(i)}\right) = \begin{cases} 11i-6, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 11i-4, & 1 \le i \le n \text{ and } i \text{ is even,} \end{cases} \quad f\left(u_4^{(i)}\right) = \begin{cases} 6, & i=1 \\ 11i-4, & 2 \le i \le n \text{ and } i \text{ is odd} \\ 11i-1, & 2 \le i \le n \text{ and } i \text{ is even,} \end{cases}$$

$$f\left(v_1^{(i)}\right) = \begin{cases} 11i-7, & 1 \le i \le n \text{ and } i \text{ is odd} \\ 11, & i=2 \\ 11i-9, & 4 \le i \le n \text{ and } i \text{ is even,} \end{cases} \quad f\left(v_2^{(i)}\right) = \begin{cases} 6i+1, & 1 \le i \le 2 \\ 11i-5, & 3 \le i \le n \text{ and } i \text{ is odd} \\ 11i-7, & 3 \le i \le n \text{ and } i \text{ is even,} \end{cases}$$

Journal of Engineering Technology
Volume 6, Issue 1, Jan. 2018, PP. 13-27
RESEARCH ARTICLE -ENGINEERNG TECHNOLOGY

$$f\left(v_3^{(i)}\right)=\begin{cases}5i+4, & 1\le i\le 2\\ 11i-3, & 3\le i\le n \text{ and } i \text{ is odd}\\ 11i-5, & 3\le i\le n \text{ and } i \text{ is even and}\end{cases}\qquad f\left(v_4^{(i)}\right)=\begin{cases}11i-1, & 1\le i\le n \text{ and } i \text{ is odd}\\ 16, & i=2\\ 11i-3, & 4\le i\le n \text{ and } i \text{ is even.}\end{cases}$$

Then the induced edge labeling f^* is obtained as follows.

$$f^*(u_iu_{i+1})=\begin{cases}14, & i=1\\ 11i, & 2\le i\le n \text{ and } i \text{ is odd}\\ 11i-1, & 2\le i\le n \text{ and } i \text{ is even,}\end{cases}\qquad f^*(v_iv_{i+1})=\begin{cases}11i-1, & 1\le i\le n \text{ and } i \text{ is odd}\\ 23, & i=2\\ 11i, & 4\le i\le n \text{ and } i \text{ is even,}\end{cases}$$

$$f^*(u_iv_i)=11i-6, \text{ for } 1\le i\le n,$$

$$f^*\left(u_iu_1^{(i)}\right)=\begin{cases}1, & i=1\\ 22, & i=3\\ 11i-10, & 5\le i\le n \text{ and } i \text{ is odd}\\ 11i-5, & 1\le i\le n \text{ and } i \text{ is even,}\end{cases}\qquad f^*\left(u_iu_2^{(i)}\right)=\begin{cases}11i-9, & 1\le i\le n \text{ and } i \text{ is odd}\\ 11i-4, & 1\le i\le n \text{ and } i \text{ is even,}\end{cases}$$

$$f^*\left(u_iu_3^{(i)}\right)=\begin{cases}11i-8, & 1\le i\le n \text{ and } i \text{ is odd}\\ 11i-3, & 1\le i\le n \text{ and } i \text{ is even,}\end{cases}\qquad f^*\left(u_iu_4^{(i)}\right)=\begin{cases}11i-7, & 1\le i\le n \text{ and } i \text{ is odd}\\ 11i-2, & 1\le i\le n \text{ and } i \text{ is even,}\end{cases}$$

$$f^*\left(v_iv_1^{(i)}\right)=\begin{cases}11i-5, & 1\le i\le n \text{ and } i \text{ is odd}\\ 11, & i=2\\ 11i-10, & 4\le i\le n \text{ and } i \text{ is even,}\end{cases}\qquad f^*\left(v_iv_2^{(i)}\right)=\begin{cases}11i-4, & 1\le i\le n \text{ and } i \text{ is odd}\\ 12, & i=2\\ 11i-9, & 4\le i\le n \text{ and } i \text{ is even,}\end{cases}$$

$$f^*\left(v_iv_3^{(i)}\right)=\begin{cases}11i-3, & 1\le i\le n \text{ and } i \text{ is odd}\\ 13, & i=2\\ 11i-8, & 4\le i\le n \text{ and } i \text{ is even and}\end{cases}\qquad f^*\left(v_iv_4^{(i)}\right)=\begin{cases}11i-2, & 1\le i\le n \text{ and } i \text{ is odd}\\ 11i-7, & 1\le i\le n \text{ and } i \text{ is even.}\end{cases}$$

Hence f is a F- root square mean labeling of $L_n \circ S_m$, for $3\le m\le 4$. Thus the graph $L_n \circ S_m$ is a F- root square mean graph, for $n\ge 2$ and $3\le m\le 4$.

Theorem 2.3. The graph P_a^b is a F- root square mean graph, for $a\ge 2$ and $b\le 3$.

Proof. Let $y_i, x_{ij1}, x_{ij2}, \ldots, x_{iji}, y_{i+1}$ be the vertices of the path P_i^j where $1\le i\le a-1$ and $1\le j\le b$.

Let $V\left(P_a^b\right)=\{y_i : 1\le i\le a\}\cup \bigcup_{i=1}^{a-1}\bigcup_{j=1}^{b}\{x_{ijk} : 1\le k\le i\}$ and

$E\left(P_a^b\right)=\bigcup_{i=1}^{a-1}\{y_ix_{ij1} : 1\le i\le b\}\cup \bigcup_{i=1}^{a-1}\bigcup_{j=1}^{b}\{x_{ijk}x_{ij(k+1)} : 1\le k\le i-1\}\cup \bigcup_{i=1}^{a-1}\{x_{iji}y_{i+1} : 1\le j\le b\}$ be the vertex set and edge set of the graph P_a^b.

Case (i). $b=2$.

Define $f : V\left(P_a^2\right)\to \{1,2,3,\ldots,(a-1)(a+2)+1\}$ as follows.

Journal of Engineering Technology
Volume 6, Issue 1, Jan. 2018, PP. 13-27
RESEARCH ARTICLE -ENGINEERNG TECHNOLOGY

$f(y_1)=1,\quad f(y_i)=(i-1)(i+2)+1,\ \ for\ 2\le i\le a,\quad f(x_{1j1})=j+1,\ \ for\ 1\le j\le 2\ \ and$

for $2\le i\le a-1,\quad f(x_{ijk})=(i-1)(i+2)+2k+j-1,\ \ for\ 1\le k\le i\ \ and\ \ 1\le j\le 2.$

Then the induced edge labeling f^* is obtained as follows.

For $1\le j\le 2,\quad f^*(y_1x_{1j1})=j\ \ and\ \ f^*(x_{1j1}y_2)=j+2.$

For $2\le i\le a-1$ and $1\le j\le 2,\quad f^*(y_ix_{ij1})=(i-1)(i+2)+j.$

For $2\le i\le a-1,\ 1\le k\le i-1,$ and $1\le j\le 2,\quad f^*(x_{ijk}x_{ij(k+1)})=(i-1)(i+2)+j+2k.$

For $2\le i\le a-1$ and $1\le j\le 2,\quad f^*(x_{iji}y_{i+1})=i(i+3)+j-2.$

Case (ii). $b=3.$

Define $f:V\left(P_a^3\right)\to\left\{1,2,3,\ldots,\frac{3(a-1)(a+2)}{2}+1\right\}$ as follows.

$f(y_1)=1,\quad f(y_2)=5,$

$f(y_i)=\frac{3(i-1)(i+2)}{2}+1,\ \ for\ 3\le i\le a,$

$f(x_{111})=2,$

$f(x_{1j1})=4j-5,\ \ for\ 2\le j\le 3,$

$$f(x_{21k})=\begin{cases}4k+5, & k=1\\ 4k+4, & k=2\end{cases}$$

$$f(x_{22k})=\begin{cases}7k+6, & k=1\\ 7k-4, & k=2\ \ and\end{cases}$$

$$f(x_{23k})=\begin{cases}5k+6, & k=1\\ 5k+4, & k=2.\end{cases}$$

For $3\le i\le a-1,\quad f(x_{ij1})=\begin{cases}\frac{3(i-1)(i+2)}{2}+j+1, & 1\le j\le 2\\ \frac{3(i-1)(i+2)}{2}+2j, & j=3\ \ and\end{cases}$

Journal of Engineering Technology
Volume 6, Issue 1, Jan. 2018, PP. 13-27

$$f(x_{ijk}) = \begin{cases} \frac{3(i-1)(i+2)}{2} + 2j + 3k - 1, & 1 \le j \le 2,\ 2 \le k \le i-1 \ and\ k\ is\ even \\ \frac{3(i-1)(i+2)}{2} + 3k - 2, & j = 3,\ 2 \le k \le i-1 \ and\ k\ is\ even \\ \frac{3(i-1)(i+2)}{2} + 2j + 3k - 3, & 1 \le j \le 3,\ 2 \le k \le i-1 \ and\ k\ is\ odd \\ \frac{3(i-1)(i+2)}{2} + +3k - 1, & j = 1,\ k = i \ and\ k\ is\ odd \\ \frac{3(i-1)(i+2)}{2} + +3k + j - 1, & j = 2,\ k = i \ and\ k\ is\ odd \\ \frac{3(i-1)(i+2)}{2} + +3k + j, & j = 3,\ k = i \ and\ k\ is\ odd \\ \frac{3(i-1)(i+2)}{2} + 3k + j, & 1 \le j \le 2,\ k = i \ and\ k\ is\ even \\ \frac{3(i-1)(i+2)}{2} + +3k - 1, & j = 3,\ k = i \ and\ k\ is\ even. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows.

$$f^*(y_1 x_{1j1}) = \begin{cases} j, & 1 \le j \le 2 \\ 5, & j = 3, \end{cases}$$

$$f^*(y_i x_{ij1}) = \begin{cases} \frac{3(i-1)(i+2)}{2} + j, & j = 1 \ and \ 2 \le i \le a-1 \\ \frac{3(i-1)(i+2)}{2} + j + 1, & j = 2 \ and \ i = 2 \\ \frac{3(i-1)(i+2)}{2} + j + 1, & j = 3 \ and \ i = 2 \\ \frac{3(i-1)(i+2)}{2} + j, & j = 2,3 \ and \ 3 \le i \le a-1, \end{cases}$$

$$f^*(x_{ij1} y_2) = \begin{cases} 3, & j = 1 \\ 2j, & 2 \le j \le 3, \end{cases}$$

$$f^*(x_{2j1} x_{2j2}) = j + 9, \quad for\ 1 \le j \le 3 \ and$$

$$f^*(x_{2j2} y_3) = \begin{cases} 14, & j = 1 \\ 13, & j = 2 \\ 15, & j = 3. \end{cases}$$

For $3 \le i \le a-1$,

$$f^*\left(x_{ijk}x_{ij(k+1)}\right)=\begin{cases}\frac{3(i-1)(i+2)}{2}+3k+2(j-1)+1, & 1\le k\le i-1 \text{ and } 1\le j\le 2\\ \frac{3(i-1)(i+2)}{2}+3k+2, & 1\le k\le i-1 \text{ and } j=3 \text{ and}\end{cases}$$

$$f^*\left(x_{iji}y_{i+1}\right)=\begin{cases}\frac{3(i-1)(i+2)}{2}+j-3, & 1\le j\le 3 \text{ and } i \text{ is odd}\\ \frac{3(i-1)(i+2)}{2}+j-2, & 1\le j\le 2 \text{ and } i \text{ is even}\\ \frac{3i(i+3)}{2}-2, & j=3 \text{ and } i \text{ is even.}\end{cases}$$

Hence f is a F- root square mean labeling of P_a^b, for $a\ge 2$ and $b\le 3$. Thus the graph P_a^b is a F- root square mean graph, for $a\ge 2$ and $b\le 3$.

Theorem 2.4. The graph D_n^* is a F- root square mean graph, for $n\ge 2$.

Proof. Let $V\left(D_n^*\right)=\{a_{i,j}:1\le i\le n,\ j=1,2,3,4\}$ and $E\left(D_n^*\right)=\{a_{i,1}a_{i+1,1}, a_{i,3}a_{i+1,3}:1\le i\le n-1\}\cup$ $\{a_{i,1}a_{i,2}, a_{i,2}a_{i,3}, a_{i,3}a_{i,4}, a_{i,4}a_{i,1}:1\le i\le n\}$ be the vertex set and edge set of the graph D_n^*.

Define $f:V\left(D_n^*\right)\to\{1,2,3,\ldots,6n-1\}$ as follows.

For $1\le i\le n$, $f(a_{i,1})=6i-4$, $f(a_{i,2})=6i-5$, $f(a_{i,3})=6i-3$ *and* $f(a_{i,4})=6i-1$.

Then the induced edge labeling f^* is obtained as follows.

For $1\le i\le n$, $f^*(a_{i,1}a_{i+1,1})=6i-1$ *and* $f(a_{i,3}a_{i+1,3})=6i$.

For $1\le i\le n$, $f^*(a_{i,1}a_{i,2})=6i-5$, $f^*(a_{i,2}a_{i,3})=6i-4$,

$f^*(a_{i,3}a_{i,4})=6i-2$ *and* $f^*(a_{i,4}a_{i,1})=6i-3$.

Hence f is a F- root square mean labeling of D_n^*. Thus the graph D_n^* is a F- root square mean graph, for $n\ge 2$.

Theorem 2.5. The latitude graph is a F- root square mean graph.

Proof. Let $V(G)=\{u_i, 1\le i\le n\}$ and $E(G)=\{u_iu_{i+1}:1\le i\le n-1\}\cup\{u_nu_1\}\cup\left\{u_iu_{n+2-i}:2\le i\le\frac{n}{2}\right\}$ be the vertex set and edge set of the latitude graph G.

A vertex labeling $f:V(G)\to\left\{1,2,3,\ldots,\frac{3n}{2}\right\}$ is defined as

Journal of Engineering Technology
Volume 6, Issue 1, Jan. 2018, PP. 13-27

$$f(u_i)=\begin{cases}3i-2, \ 1\le i\le \frac{n}{2}\\ \frac{3n}{2}, \ 1\le i\le \frac{n}{2}+1\\ 3n-3i+3, \ \frac{n}{2}+2\le i\le n-1\\ 2, \ i=n.\end{cases}$$

Then the induced edge labeling f^* is obtained as follows.

$$f^*(u_iu_{i+1})=\begin{cases}3i-1, \ 1\le i\le \frac{n}{2}\\ \frac{3n}{2}-2, \quad i=\frac{n}{2}+1\\ 3n-3i+1, \quad \frac{n}{2}+2\le i\le n-1,\end{cases}$$

$f^*(u_nu_1)=1$ *and* $f^*(u_iu_{n+2-i})=3i-3.$ *for* $2\le i\le \frac{n}{2}.$

Hence f is a F- root square mean labeling of latitude graph. Thus the latitude graph is a F- root square mean graph.

3. F-root square meanness of some path related graphs

Theorem 3.1. The graph $P_n(X_1, X_2, \ldots, X_n)$ is a F- root square mean graph, for $1\le X_i\le 3$ and $|X_i - X_{i+1}|\le 1,$ for $1\le i\le n.$

Proof. Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n. Let $v_i^{(1)}, v_i^{(2)}, v_i^{(3)}, \ldots, v_i^{(X_i)}$ be the pendant vertices attached at u_i, for $1\le i\le n.$

Define $f: V(P_n(X_1, X_2, \ldots, X_n))\to \left\{1,2,3,\ldots, \sum_{i=1}^{n} X_i + n\right\}$ as follows.

$$f\left(v_i^{(1)}\right)=\begin{cases}2, \quad X_i=1\\ 1, \quad X_i\ne 1.\end{cases}$$

For $2\le i\le n,$ $$f\left(v_i^{(1)}\right)=\begin{cases}\sum_{i=1}^{n} X_i + i, \quad X_i=2,3\\ \sum_{i=1}^{n} X_i + i+1, \quad X_i=1.\end{cases}$$

Journal of Engineering Technology
Volume 6, Issue 1, Jan. 2018, PP. 13-27
RESEARCH ARTICLE -ENGINEERNG TECHNOLOGY

For $1 \le i \le n,\quad f\left(v_i^{(j)}\right) = \begin{cases} f\left(v_i^{(1)}\right)+2, & j=2 \\ f\left(v_i^{(1)}\right)+3, & X_i = 3 \text{ and } j=3 \text{ and} \end{cases}$

$$f(u_i) = \begin{cases} f\left(v_i^{(1)}\right)+1, & X_i = 2,3 \\ f\left(v_i^{(1)}\right)-1, & X_i = 1. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows.

For $1 \le i \le n-1,\quad f(u_iu_{i+1}) = \begin{cases} f(u_i)+1, & X_i = 1,2 \\ f(u_i)+2, & X_i = 3 \text{ and} \end{cases} \qquad f^*\left(v_1^{(1)}u_1\right) = 1.$

For $1 \le i \le n,\quad f^*\left(v_i^{(1)}u_i\right) = \begin{cases} f\left(v_i^{(1)}\right)+1, & X_i = 2,3 \\ f\left(v_i^{(1)}\right)-1, & X_i = 1 \text{ and} \end{cases} \qquad f^*\left(v_i^{(j)}u_i\right) = \begin{cases} f(u_i), & X_i = 2,3 \text{ and } j=2 \\ f(u_i)+1, & X_i = 3 \text{ and } j=3. \end{cases}$

Hence f is a F- root square mean labeling of $P_n(X_1, X_2, \ldots, X_n)$. Thus the graph $P_n(X_1, X_2, \ldots, X_n)$ is a F- root square mean graph, for $1 \le X_i \le 3$ and $|X_i - X_{i+1}| \le 1$, for $1 \le i \le n$.

Theorem 3.2. The splitting graph $S'(P_n)$ is a F- root square mean graph, for $n \ge 2$.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of the path P_n. Let $v_1, v_2, \ldots, v_n, v'_1, v'_2, \ldots, v'_n$ be the vertices of the graph $S'(P_n)$. Let $V(S'(P_n)) = \{v_i, v'_i : 1 \le i \le n\}$ and $E(S'(P_n)) = \{v_iv_{i+1}, v_iv'_{i+1}, v'_iv_{i+1} : 1 \le i \le n-1\}$ be the vertex set and edge set of the splitting graph $S'(P_n)$.

Case (i). n is odd.

Define $f : V(S'(P_n)) \to \{1, 2, 3, \ldots, 3n-2\}$ as follows.

$f(v_i) = \begin{cases} 4i-3, & 1 \le i \le 2 \\ 3. & i = 3 \\ 3i-4, & 4 \le i \le n \text{ and } i \text{ is odd} \\ 3i, & 4 \le i \le n \text{ and } i \text{ is even and} \end{cases} \qquad f(v'_i) = \begin{cases} 6, & i=1 \\ 2. & i=2 \\ 3i-2, & 3 \le i \le n. \end{cases}$

Then the induced edge labeling f^* is obtained as follows.

$f^*(v_iv_{i+1}) = \begin{cases} i+2, & 1 \le i \le 2 \\ 3i-1, & 3 \le i \le n-1, \end{cases} \qquad f^*(v_iv'_{i+1}) = \begin{cases} 5i-4, & 1 \le i \le 2 \\ 3i-2, & 3 \le i \le n-1 \text{ and } i \text{ is odd} \\ 3i, & 3 \le i \le n-1 \text{ and } i \text{ is even and} \end{cases}$

$$f^*(v'_iv_{i+1}) = \begin{cases} 5, & i=1 \\ 2, & i=2 \\ 3i, & 3 \le i \le n-1 \text{ and } i \text{ is odd} \\ 3i-2, & 3 \le i \le n-1 \text{ and } i \text{ is even.} \end{cases}$$

Journal of Engineering Technology
Volume 6, Issue 1, Jan. 2018, PP. 13-27
RESEARCH ARTICLE -ENGINEERNG TECHNOLOGY

Case (ii). n is even.

Define $f : V(S'(P_n)) \to \{1,2,3,\ldots,3n-2\}$ as follows.

$$f(v_i) = \begin{cases} 4-i, & 1 \le i \le 2 \\ 3i-1, & 3 \le i \le n \text{ and } i \text{ is odd} \\ 3i-3, & 3 \le i \le n \text{ and } i \text{ is even and} \end{cases} \qquad f(v_i') = \begin{cases} 1, & i=1 \\ 3i-3, & 2 \le i \le n \text{ and } i \text{ is odd} \\ 3i-2, & 2 \le i \le n \text{ and } i \text{ is even.} \end{cases}$$

Then the induced edge labeling f^* is obtained as follows.

$$f^*(v_i v_{i+1}) = 3i-1, \text{ for } 3 \le i \le n-1,$$

$$f^*(v_i v_{i+1}') = \begin{cases} 3i, & 3 \le i \le n-1 \text{ and } i \text{ is odd} \\ 3i-2, & 3 \le i \le n-1 \text{ and } i \text{ is even and} \end{cases}$$

$$f^*(v_i' v_{i+1}) = \begin{cases} 3i-2, & 1 \le i \le n-1 \text{ and } i \text{ is odd} \\ 3i, & 1 \le i \le n-1 \text{ and } i \text{ is even.} \end{cases}$$

Hence f is a F- root square mean labeling of $S'(P_n)$. Thus the splitting graph $S'(P_n)$ is a F- root square mean graph, for $n \ge 2$.

Theorem 3.3. The graph $P(1,2,\ldots,n-1)$ is a F- root square mean graph, for $n \ge 2$.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of the path P_n and let u_{ij} be the vertices of the partition of K_{2,m_i}, with cardinality m_i, $1 \le i \le n-1$ and $1 \le j \le m_i$.

Define $f : V(P(1,2,\ldots,n-1)) \to \{1,2,3,\ldots,n(n-1)+1\}$ as follows.

$f(v_i) = i(i-1)+1$, for $1 \le j \le n$ and

$f(u_{ij}) = i(i-1)+2j$, for $1 \le j \le i$ and $1 \le j \le n-1$.

Then the induced edge labeling f^* is obtained as follows.

$f^*(v_i u_{ij}) = i(i-1)+j$, for $1 \le j \le i$ and $1 \le i \le n-1$,

$f^*(u_{ij} v_{i+1}) = i^2 + j$, for $1 \le j \le i$ and $1 \le j \le n-1$.

Hence f is a F- Root square mean labeling of $P(1,2,\ldots,n-1)$. Thus the graph $P(1,2,\ldots,n-1)$ is a F- Root square mean graph, for $n \ge 2$.

4. Conclusion

In this paper, the F- root square meanness property of the planar grid $P_m \times P_n$, for $m \le 5$, the graph $L_n \circ S_m$, for $3 \le m \le 4$, the graph P_a^b for $a \ge 2$ and $b \le 3$, the graph D_n^*, the latitude graph, the graph $P_n(X_1, X_2, \ldots, X_n)$, the splitting graph $S'(P_n)$ and the arbitrary supersubdivision graph $P(m_1, m_2, \ldots, m_{n-1})$ of the path P_n are studied. It is possible to investigate the F- root square meanness for several operations in graphs.

References

[1]. S. Arockiaraj , A. Durai Baskar and A. Rajesh Kannan, "F-Root Square Mean Labeling of Graphs obtained from paths", International Journal of Mathematical Combinatorics, 2(2017),92-104.

[2]. S. Arockiaraj , A. Durai Baskar and A. Rajesh Kannan, "F-root square mean labeling of some cycle related graphs", Journal of Engineering Technology, 6(2)(2017),440-448.

[3]. S. Arockiaraj , A. Durai Baskar and A. Rajesh Kannan, "F-Root Square Mean Labeling of some ladder related graphs", Global Journal of Pure and Applied Mathematics, 13(2)(2017),281-289.

[4]. S. Arockiaraj , A. Durai Baskar and A. Rajesh Kannan, "F-Root Square Mean Labeling in the context of duplication of graph elements", International Journal of Pure and Applied Mathematics, 117(6)(2017), 59-69.

[5]. A. Durai Baskar, S. Arockiaraj and B.Rajendran, "F-Geometric Mean labeling of some chain graphs and thorn graphs", Kragujevac Journal of Mathematics, 37(1)(2013), 163-186.

[6]. A. Durai Baskar, S. Arockiaraj and B.Rajendran, "Geometric Mean Labeling of graphs obtained from some graph operations", International Journal of Mathematical Combinatorics, 1(2013),85-98.

[7]. A. Durai Baskar and S. Arockiaraj, "F-Geometric Mean Graphs", International Journal of Applications and Applied Mathematics, 10(2)(2015),937-952.

[8]. A. Durai Baskar, S. Arockiaraj and B.Rajendran, "Geometric Meanness of graphs obtained from paths", Utilitas Mathematica, 101(2016),45-68.

[9]. J.A. Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, 17 (2016), #DS6.

[10]. F.Harary, Graph Theory, Narosa Publishing House Reading, New Delhi (1988).

[11]. K.M. Kathiresan and S.Amutha, "Arbitrary supersubdivision of stars are graceful", Indian Journal of Pure and Applied Mathematics, 35(1)(2004)81-84.

[12]. P.Lawrence Rozario Raj and S.Koilraj, "Cordial labeling for the splitting graph of some standard graphs", International Journal of Mathematics and Soft Computing, 1(1)(2011)105-114.

[13]. T.K.Mathew Varkey and B.S. Sunoj, "Some new families of odd graceful graphs", International Journal of Mathematical Combinatorics, 3(2016),158-161.

[14]. A.Meenakumari and S.Arockiaraj, "On (1,1,0)- F- Face mean labeling of some graphs", Utilitas Mathematica, 103(2017),139-159.

[15]. S.S. Sandhya, S. Somasundram and S. Anusa, "Root Square Mean Labeling of graphs", International Journal of Contemporary Mathematical Sciences, 9(14)(2014),667-676.

[16]. R.Vasuki and A.Nagarajan, "Odd mean labeling of the graphs $P_{a,b}$, P_a^b and $P_{<2a>}^b$ ", Kragujevac Journal of Mathematics, 36(1)(2012), 141-150.

[17]. R.Vasuki, A.Nagarajan and S.Arockiaraj, "Even vertex odd mean labeling of graphs", SUT Journal of Mathematics, 49 (2)(2013) ,79-92.