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Performance Analysis of MIMO System under Fading Channels (Rayleigh & Rician) Using SVD, PCA& FSVD

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Abstract: Nowadays wireless channels effecting due to fading is the main issue in communications. MIMO techniques enable higher spectral efficiency and improve robustness against channel fading. In this paper an effort has been made to illustrate the performance comparison of the Rayleigh & Rician fading channel models for MIMO system using MATLAB simulation. The Water filling algorithm has been implemented for allocating the power to the MIMO channels for enhancing the capacity of MIMO system comparing in two different fading channels. This paper proposes to compare the performance of MIMO system using Principal component analysis (PCA), Singular value decomposition (SVD), & Fractional order singular value decomposition (FSVD) along with water filling algorithm for each in two different fading channels (Rayleigh & Rician). This work provides the quantitative results for comparison of the Capacity Vs SNR performance for SISO & MIMO systems and also comparison of probability density function (PDF) of different MIMO system. The simulation study has been carried out to reflect the effectiveness of the proposed work. It is identified that the MIMO system performance can be better in Rician fading channel using FSVD.

Keywords: MIMO system, Principal component analysis, Singular value decomposition, fractional order singular value decomposition, water filling algorithm

1. Introduction:

The goal of future wireless communications systems is to provide a wide variety of high quality high-rate services with minimum requirements on spectrum, power consumption and hardware complexity. MIMO systems where multiple antennas are deployed at both transmitter and receiver have the attracted attention in next generation wireless communication. It has rapidly gained in popularity over past decade due to its powerful performance-enhancing capabilities [1]. In MIMO system, the multipath is used as a benefit. With MIMO system the capacity of channel can be increase as it deploys multiple antennas at both ends. MIMO techniques enable higher spectral efficiency and improve robustness against channel fading. Among MIMO techniques, the closed-loop MIMO can provide extra gain compared to the open-loop MIMO, thanks to the provision of the channel state information at the transmitter (CSIT) [2,3]. The promise of a high performance return from using MIMO systems largely relies on the assumption of perfect coherent reception, i.e., perfect channel state information at the receiver (CSIR), and even perfect CSIT with some designs [4].

The capacity of MIMO system can further be increased, if the channel parameters are known at both transmitter and receiver by allocating extra power at the transmitter using water filling algorithm to all the channels. If the channel parameters are unknown at the transmitters and known at the receivers, equal power is allocated to each transmitter and estimates the capacity using water filling algorithm. Water filling is solution of several optimization problems related to channel capacity. The well known water filling algorithm solves the problem of maximizing the mutual information between the input and output of a channel [5, 6].

Rest of the paper is organized by comparing the performance of MIMO system in both Rayleigh and Rician fading channels using principal component analysis, singular value decomposition, and fractional order singular value decomposition including water filling algorithm for each as follows, section 2 describes the introduced models i.e. MIMO, Rayleigh and Ricean fading models. Section 3 describes techniques used for improving capacity i.e. PCA, SVD, FSVD. Section 4, 5, 6 provides MIMO capacity results for PCA, SVD, and FSVD. In Section 7 comparative experimental results are shown and Conclusions are outlined in Section 8.

2. Introduced Models:

2.1 Rayleigh Channel:

The effects of multipath embrace constructive and destructive interference, and phase shifting of the signal. This causes Rayleigh fading. There is no line of sight (NLOS) path means no direct path between transmitter and receiver in Rayleigh fading channel [3]. The received signal can be simplified to:

$$R(n)=h_n,\tau S_{n-m}+w(n) \qquad (1)$$

Where w(n) is AWGN noise with zero mean and unit variance, h(n) is channel impulse response i.e.

$$h(n)=\alpha(n)e^{-j\theta}(n) \qquad (2)$$

Where $\alpha(n)$ and $e^{-j\theta}(n)$ are attenuation and phase shift for nth path.

If the coherence bandwidth of the channel is larger than signal bandwidth, the channel is called flat; otherwise it is frequency-selective fading channel. In this paper, MIMO is simulated under frequency-selective fading channel. The Rayleigh distribution [7] is basically the magnitude of the sum of two equal independent orthogonal Gaussian random variables and the probability density function (pdf) given by:

$$P(Z)= Z/\sigma^2 * e^{-Z_2/2\sigma_2} \qquad (3)$$

Where σ^2 is the time-average power of the received signal and eq.(3) is called a Rayleigh random variable.

2.2 Rician Channel:

In environments where there is a dominant Line-of-Sight (LOS) path between the transmitter and the receiver, the complex Gaussian distributed fading coefficient should be modelled with a non-zero mean, giving rise to the Rician fading. Or also say that, Rayleigh fading with strong line of sight (LOS) content is said to have a Rician distribution, or to be Rician fading.

The Rician distribution is usually characterized by the Rician factor k.

$$K = 2\sigma^2 \qquad (4)$$

This shows the relative strength of the direct LOS path component of the fading coefficient. When k=0, this model reduces to Rayleigh model and k= ∞ the fading becomes deterministic giving grow to an AWGN channel [8, 9].

3. Description on SVD, PCA and FSVD:

3.1 Singular value decomposition:

The SVD technique decouples the channel matrix in spatial domain in a way similar to the DFT decoupling the channel in the frequency domain. The channel matrix H is the T x R channel matrix. If H has independent rows and columns, SVD yields:

$$\mathrm{H} = \mathrm{U}\sum\mathrm{V}^{\mathrm{h}} \qquad (5)$$

Where U and V are unitary matrices and V^h is the hermitian of V. U has dimension of R x R and V has dimension of T x T. Σ is a T x R matrix. If T = R, then Σ become a diagonal matrix. If T > R, it is made of R x R diagonal matrix followed by T – R zero columns. If T < R, it is made of T x T diagonal matrix followed by R – T zero rows. This operation is called the singular value decomposition of H [12, 13].

In case, where T ≠ R, the number of spatial channels become restricted to the minimum of T and R. If the number of transmit antennas is greater than the receive antennas (T > R), U will be an R x R matrix, V will be a T x T matrix and **Σ** will be made of a square matrix of order R followed by T-R zero columns [12.13].

3.2 Principal component analysis:

Principal Component Analysis (PCA) [10] is the general name for a technique which uses sophisticated underlying mathematical principles to transforms a number of possibly correlated variables into a smaller number of variables called principal components. The origins of PCA lie in multivariate data analysis; however, it has a wide range of other applications. PCA has been called, 'one of the most important results from applied linear algebra '[11] and perhaps its most common use is as the first step in trying to analyze large data sets.

Principal component analysis defines independence by considering the variance of the data in the original basis. It seeks to de-correlate the original data by finding the directions in which variance is maximized and then use these directions to define the new basis. Recall the definition for the variance of a random variable, Z with mean, μ.

$$\sigma_2{}^2 = E\,[(Z- \mu)^2] \qquad (6)$$

3.3 Fractional order singular value decomposition:

The FSVD technique developed by using SVD technique where FSVD is differs by SVD with alpha factor. Now the H matrix representation using FSVD can be given as

$$H = US^{\alpha}V^{h} \qquad (7)$$

Where U, S and V are the corresponding matrices in FSVD, and α is the fractional parameter satisfies $0 \le \alpha \le 1$.

3.4 Water filling algorithm:

Water filling refers to a technique whereby the power for the spatial channels are adjusted based on the channels gain. The channel with high gain and signal to noise ratio (SNR) is given more power. More power maximizes the sum of data rates in all sub channels. The data rate in each sub channel is related to the power allocation by Shannon's Gaussian capacity formula C = B log (1 + SNR). However, because of the capacity is a logarithmic function of power, the data rate is usually insensitive to the exact power allocation. This motivates the search for simpler power allocation schemes that can perform close to the optimal.

4. Results for SVD In Rayleigh & Ricean Fading Models:

4.1 MIMO capacity results for SVD in Rayleigh fading model:

The referred literature regarding performance evaluation of MIMO system using SVD [16, 17] proposed the system only with the Rayleigh model. Using those literatures as reference the results are as below:

Figure 1 shows the capacity versus signal to noise ratio plot for MIMO system using SVD in Rayleigh fading channel. In previous literature [17], it is provided that the MIMO system itself will improve the capacity over channels by employing SVD in 4X4 MIMO model. The proposed paper employed singular value decomposition to increase the capacity even further by increasing the number of elements of transmitter and receiver to 8X8 MIMO model. And the plot shows the comparative capacity results for SISO and MIMO systems.

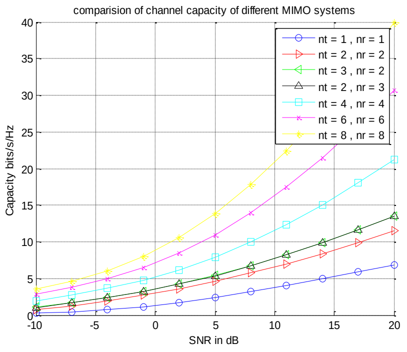

Fig 1

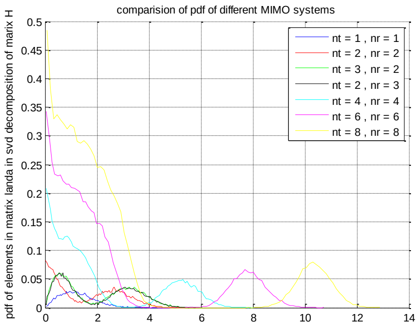

Fig 2

Figure 1: comparison of channel capacity of MIMO system Using SVD in Rayleigh model;
Figure 2: comparison of PDF of different MIMO systems using SVD in Rayleigh model

Figure 2 shows the plot for comparison of PDF of different MIMO systems in Rayleigh fading channel, which provides the information regarding histogram for the system to estimate the power, employed for each channel, and can be decided by the technique water filling algorithm.

4.2 MIMO capacity results for SVD in Rician fading model:

Rician fading model can be described in section 2.2 and section 4.2. Here in this section the results for the capacity of MIMO system using SVD in Rician fading channel as shown below:

Figures 3 and 4 depicts the plots for the comparative capacity of SISO and MIMO systems using SVD in Rician fading channel and the probability density function of different MIMO systems in Rician model respectively for k=15.

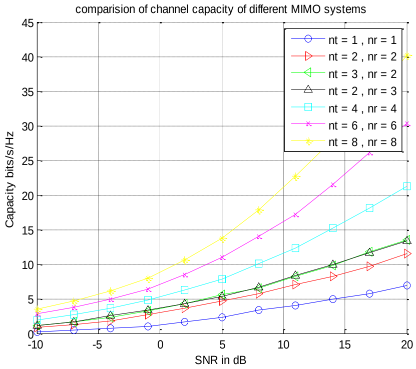

Fig 3

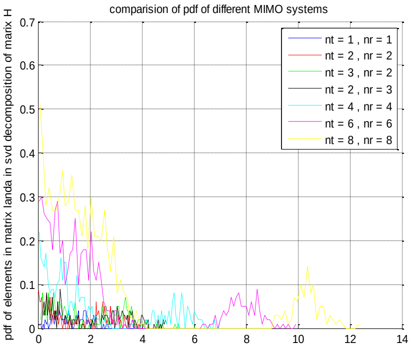

Fig 4

Figure 3: comparison of channel capacity

Figure 4: comparison of PDF of different of MIMO system using SVD in Rician fading channel

5. Results for PCA in Rayleigh & Ricean Fading Models:

5.1 MIMO capacity results for PCA in Rayleigh fading channel:

Most of the referred literature [14] and other existing literature are centred on the channel estimation in MIMO system using principal component analysis (PCA). However, there is a lack of studies of both effects on MIMO capacity, and having working system at low signal to noise ratio have been published. The work in [14], the PCA, which is not prevalent in channel estimation, and its adaptation to channel information are provided. However in the work referred above, a study of performance of MIMO capacity in different fading channels has not been found, and it is presented in this work.

Figure 5 shows the capacity versus signal to noise ratio for Rayleigh fading channel using PCA in MIMO system. This plot includes SISO, and MIMO system comparison as to mention MIMO system will give better performance while comparing to SISO (Single input and single output system) and the external system PCA added to demonstrate the performance of SISO and MIMO systems in Rayleigh fading channel.

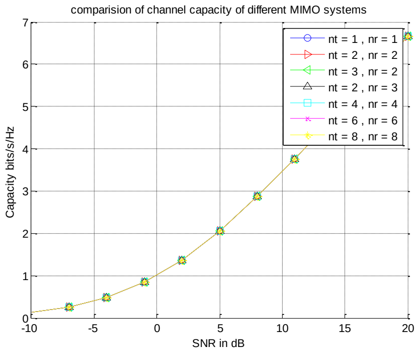

Figure 5: comparison of channel capacity of MIMO system using PCA in Rayleigh model

5.2 MIMO capacity results for PCA in Rician fading channel:

Rician fading occurs when one of the paths, typically a line of sight signal, is much stronger than the others. In [14], there is no assumption of Rician fading model to compare with, but to enhance the knowledge on fading models and performance of MIMO system in different fading models we interested in analyzing this. A semi analytical expression is presented for the probability density function and in [15]. However, in the literature, up to the knowledge on Rician fading channel we introduced the performance in capacity of MIMO system using PCA. And the results are as shown below for k=15 in figure 6.

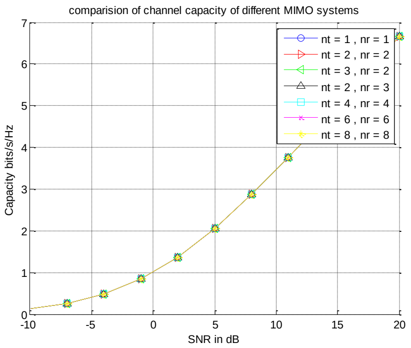

Figure 6: comparison of channel capacity of MIMO system using PCA in Rician model

6. Results for FSVD in Rayleigh & Ricean Fading Models:

6.1 MIMO capacity results for FSVD in Rayleigh fading model:

To alleviate the capacity of MIMO system more extent, the proposed paper introduces another technique called Fractional order singular value decomposition (FSVD) for improving capacity of MIMO system. Now FSVD B is defined as

$$B=U\,S^{\alpha}\,V^{T} \qquad (8)$$

Where U, S and V are the corresponding matrices, and in order to achieve the above underlying ideas, α is a fractional parameter that satisfies $0\leq\alpha\leq1$.

Figure 7 shows the capacity versus signal to noise ratio plots for MIMO system using fractional order singular value decomposition and probability density function within the value of alpha between0 to 1.

For alpha=0.4:

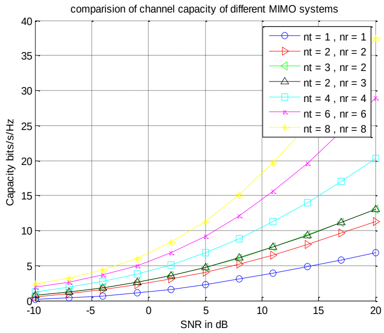

Fig (a)

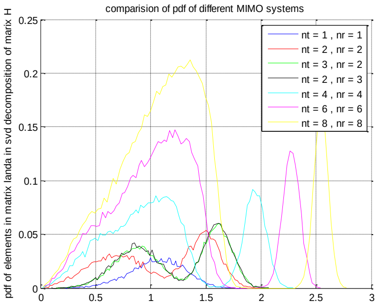

Fig (b)

For alpha=0.6:

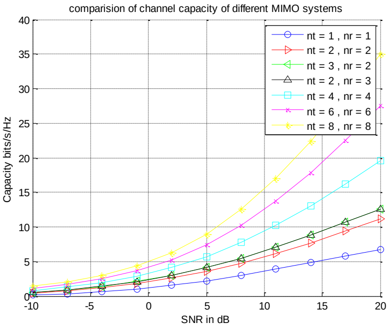

Fig (c)

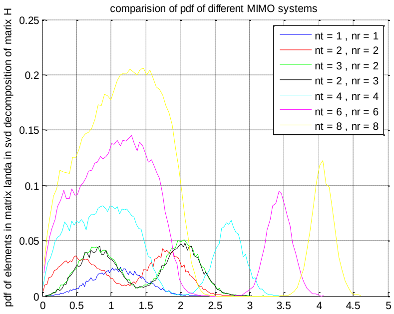

Fig (d)

For alpha=0.8

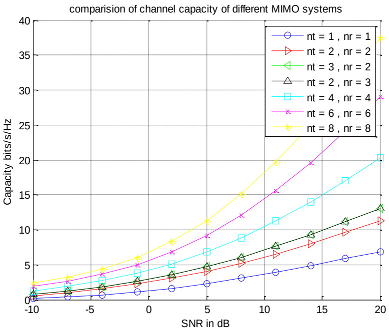

Fig (e)

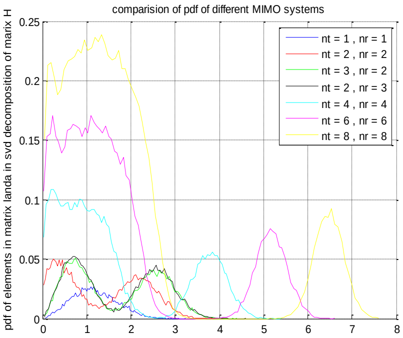

Fig (f)

Figure 7: comparison of channel capacity and PDF of MIMO system using FSVD in Rayleigh channel (a, b) for α=0.4, (c, d) for α=0.6, (e, f) for α=0.8

6.2 MIMO Capacity results for FSVD in Rician fading model:

In this section the introduced fading channel is Rician fading channel. We will analyze by employing Rician channel, the performance of MIMO system using FSVD for alpha value between 0 & 1.

For alpha=0.4:

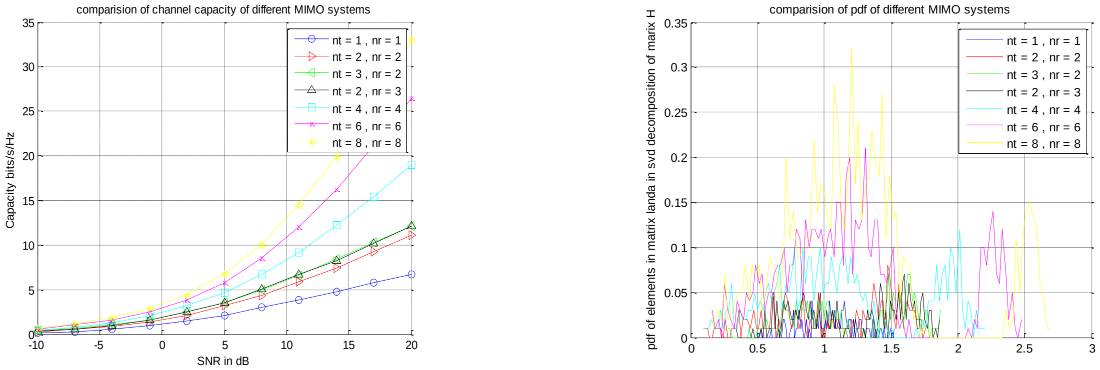

Fig (a) Fig (b)

For alpha=0.6:

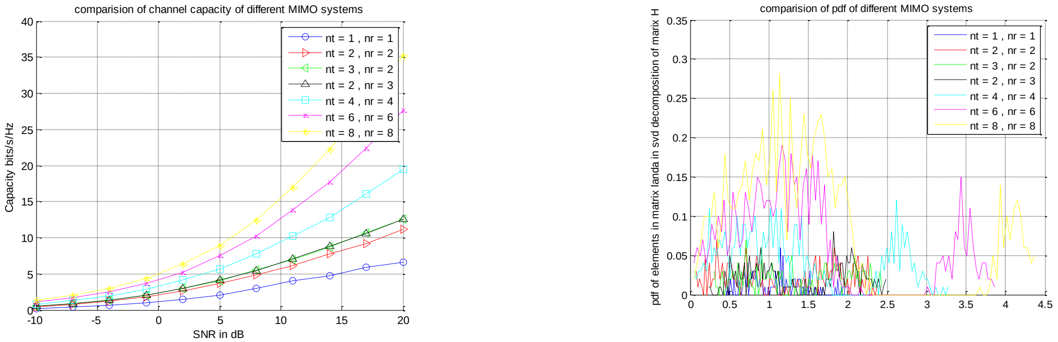

Fig (c) Fig (d)

For alpha=0.8:

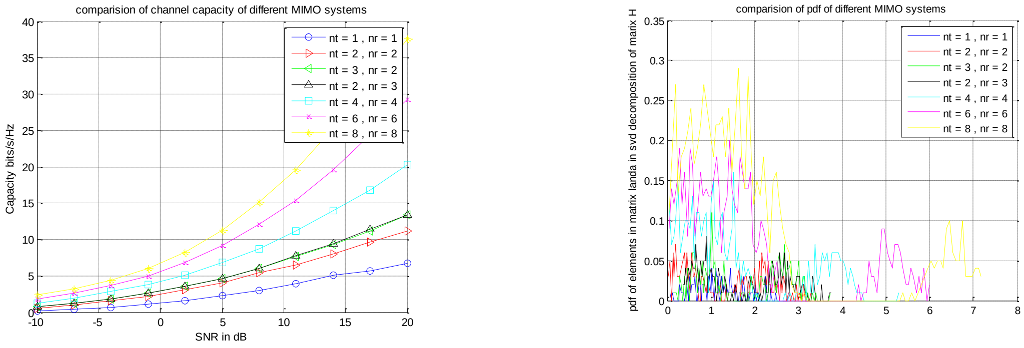

Fig (e) Fig (f)

Figure 8: comparison of channel capacity and PDF of MIMO system using FSVD in Rician channel (a, b) for α=0.4, (c, d) for α=0.6, (e, f) for α=0.8

7. Comparing Experimental Results:

This paper has been introduced with results through several sections till now; in this section we can have a clear understanding on variation of capacity in MIMO system. Basically by increasing the number of antennas at transmitter and receiver section i.e. by employing multiple input multiple output (MIMO) arrangement, the performance of the system increases. The comparison of SISO and MIMO systems (1T$_x$ X 1R$_x$; 2X2; 3X2; 2X3; 4X4; 6X6; 8X8) in two different fading channels using water filling algorithm has been introduced. The considered fading channels are Rayleigh fading channel and Rician fading channel. In each fading channel the performance of MIMO system in PCA, SVD, and FSVD has been analyzed. In previous sections the results are shown in graphs, now in this section the numerical results for each technique applied to the MIMO system has been presented in a table.

Tabular form for capacity variations in Rayleigh and Rician fading models used in MIMO system of 8X8 antenna arrangement given below.

Table 1: comparison of capacity variations in MIMO system

Rayleigh fading channel			Rician fading channel		
SVD	PCA	FSVD (α=0.8)	SVD	PCA	FSVD (α=0.8)
0.137	2.845	3.523	0.569	3.631	4.123
0.262	3.513	4.321	0.832	4.423	5.116
0.483	5.140	6.539	0.914	6.290	7.263
0.843	7.931	8.912	1.235	8.862	9.897
1.370	11.482	12.661	1.416	12.561	13.739
2.057	14.122	15.916	2.932	15.923	16.902
2.869	20.423	21.632	3.659	21.591	22.716
3.764	22.585	24.245	4.353	24.021	25.917

In the above tabular results of capacity variations in MIMO system, for FSVD α=0.8, whose capacity is compared with other techniques gives best results in MIMO system using RICIAN fading channel.

8. Conclusion & Future Scope:

In this paper, the capacity behaviour of MIMO system has been compared in Rayleigh and Rician fading channels using three different techniques named principal component analysis (PCA), Singular value decomposition (SVD) and Fractional order Singular value decomposition (FSVD). Showing a wide variety of results of these systems, depending on the number of elements and the SNR, these results show up which technique gives the best results for which fading channel by increasing the number of elements in MIMO system.

The simulation results gives best results for FSVD at $\alpha=0.8$ in Rician fading channel, where the number of antenna elements are arranged in 8X8 MIMO system. For future work the K factor for Rician channel can be varied and still the better results for the capacity can find further.

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